Constraining primordial non-Gaussianity via multi-tracer technique with Euclid and SKA

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Main Message

We can test the extremely small primordial non-Gaussianity at the level of $\sigma(f_{NL})=O(0.1)$ with Euclid and Square Kilometre Array (SKA).
What’s Primordial non-Gaussianity?

- Non-Gaussian initial fluctuations arise in several scenarios of inflation.

\[ \Phi = \phi_G + f_{NL} \left( \phi_G^2 - \langle \phi_G^2 \rangle \right) \]

- Even the simplest model predicts small but non-vanishing $f_{NL}$ of $O(0.01)$.

- PNG has primarily been constrained from the bispectrum in CMB temperature fluctuations.

- WMAP: $\sigma(f_{NL}) < 100$ [Bennet+, 2013]
- Planck: $\sigma(f_{NL}) < 10$ [Planck collaboration, 2013]
- Ideal: $\sigma(f_{NL}) \sim 3$ [Komatsu+Spergel, 2001]
Luminous sources such as galaxies must be most obvious tracers of the large scale structure.

The galaxy density contrast $\delta_{\text{gal}}$ is linearly related to the underlying dark matter density contrast $\delta_{\text{DM}}$ though the bias $b_h$:

$$\delta_{\text{gal}}(M, z, k) = b_h(M, z, k)\delta_{\text{DM}}(z, k)$$

In the Gaussian case, the bias is scale-invariant: $b_h = b_h(M, z)$. 
PNG in Large Scale Structure

- Primordial non-Gaussianity induces the scale dependent-bias such that the effect dominates at very large scales:

\[ \Delta b = \frac{2f_{\text{NL}} \delta_c}{MD_+} (b_L - 1) - \frac{1}{\delta_c} \frac{d}{d \ln \nu} \left( \frac{dn/dM}{dn_G/dM} \right) \]

- [Dalal+(2008), Desjacques+(2009)]

- Galaxy surveys can effectively constrain \( f_{\text{NL}} \) to the level comparable to CMB temp. anisotropies.
$$P_{\text{galaxy}} = (b_G + \Delta b)^2 P_{\text{DM}}$$
Accessing ultra-large scales

- Clustering analysis at large scales are limited due to cosmic variance.

MULTI-TRACER TECHNIQUE [Seljak (2009)]

- a method to reduce the cosmic variance using multiple tracers with different biases.
- The availability of multiple tracers allows significantly improved statistical error in the measurement of $f_{NL}$. 
Multi-tracer technique

[Seljak (2009)]

\[ \sigma(f_{\text{NL}}) = O(10) \]

\[ \sigma(f_{\text{NL}}) < 1 ! \]

✓ If we treat the data as the single group, the galaxy survey can constrain \( f_{\text{NL}} \) to the level comparable to CMB:

Full galaxy samples

Splitting the data into subsamples...

Group 1  Group 2  Group 3  ...
Multi-tracer technique

- Angular power spectra
- Accuracy for $b_2/b_1$

\[
\sigma \left( \frac{b_2}{b_1} \right) \propto \sqrt{N_1^{-1} + N_2^{-1}} \quad (N_1, N_2 \gg 1)
\]

We can make a measurement of the ratio of two biases that is only limited by shot noise and hence beats cosmic variance!
The accuracy of the amplitude itself is limited by CV, but for the ratio between the powers there is NO fundamental limit!
Survey design

- **Optical/infrared photometric survey : Euclid**
  - Covers 15,000 [deg$^2$].
  - Provides **redshift information** via photometric redshifts.
  - We use various galaxy properties to infer the halo mass.

- **Radio continuum survey : SKA phase-1/2**
  - Covers 30,000 [deg$^2$] out to high-z.
  - The redshift information is **not available**.
  - Halo mass can be estimated from the galaxy type.
  
  [Ferramacho+ (2014)]

- **SKA+Euclid : 9,000 [deg$^2$]**
Fisher matrix analysis

\[
F_{\alpha\beta} = \sum_{\ell = \ell_{\text{min}}}^{\ell_{\text{max}}} \sum_{I,J} \frac{\partial C_I(\ell)}{\partial \theta^\alpha} \left[ \text{Cov}(C(\ell), C(\ell)) \right]^{-1} \frac{\partial C_J(\ell)}{\partial \theta^\beta}
\]

✓ Covariant matrix generalized to multiple tracers with different sky areas with some overlap:

\[
\text{Cov}\left[ C_{i(bb')}(\ell), C_{j(b\bar{b}')}(\ell') \right] = \frac{\delta^K_{i,j} \delta^K_{\ell\ell'}}{(2\ell + 1) \Delta \ell} \frac{4\pi \Omega_w^{(bb'\bar{b}\bar{b}')}}{\Omega_w^{(bb')} \Omega_w^{(\bar{b}\bar{b}')}} \left[ C_{i(b\bar{b})}(\ell) C_{i(b\bar{b}')}(\ell) + C_{i(b\bar{b}')}(\ell) C_{i(b\bar{b})}(\ell) \right]
\]

Effect of different sky areas

[DY+Takahashi+Oguri (2014)]
The constraining power increases with $N_M$.
Even 2-tracers drastically improve the constraint.

- Combining multiple z-bins improves substantially $\sigma(f_{NL})$.
- Galaxy samples as far as $z=3.2$ contribute to the constraint.
- Realistic: $z_{\text{max}}=2.7 \rightarrow \sigma(f_{NL})=0.66$

[DY+Takahashi+Oguri (2014)]
The constraints of $\sigma(f_{NL})=O(1)$ can be obtained even with a single survey. Combining Euclid and SKA, even stronger constraints of $\sigma(f_{NL})=O(0.1)$ can be obtained.
Summary

- Splitting the galaxy samples into the subsamples by the inferred halo mass and redshift, constraints on $f_{\text{NL}}$ drastically improve.

- The constraints of $\sigma(f_{\text{NL}}) = O(1)$ can be obtained even with a single survey. Combining Euclid and SKA, even stronger constraints of $\sigma(f_{\text{NL}}) = O(0.1)$ can be obtained.

Thank you!
Efficiency of mass-binning

- Nonvanishing overlap leads to improved constraints on $f_{NL}$, which becomes smallest in the case of maximal overlap.

- There is a critical value of mass ratio which results in the tightest constraint.

The tightest constraint would be obtained when the two shot noises becomes comparable.

(Changing the value of the mass ratio leads to the larger shot noise for one of the mass-bins and smaller for the other.)
Number density of galaxies

**SKA1**

**SKA2**

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**Graphs:**

- **SKA1**
  - dN/dz [steradian\(^{-1}\)]
  - Red line: SFG
  - Orange line: RQQ
  - Blue line: FRI
  - Green line: SB
  - Light blue line: FRII

- **SKA2**
  - dN/dz [steradian\(^{-1}\)]
  - Red line: SFG
  - Orange line: RQQ
  - Blue line: FRI
  - Green line: SB
  - Light blue line: FRII

**Axes:**
- **x-axis:** redshift \(z\)
- **y-axis:** dN/dz [steradian\(^{-1}\)]
Angular power spectra

\[ C_{i(bb')}(\ell) = \int_0^\infty dz W_{i(b)} W_{i(b')} \frac{H}{\chi^2} P_\delta \left( \frac{\ell + \frac{1}{2}}{\chi}, z \right) \]

- \( W_{i(b)} \): window fn

\[ W_{i(b)} = \frac{1}{\bar{N}_{i(b)}} \frac{d^2V}{dz d\Omega} \int_0^\infty dM \frac{dn}{dM} S_{i(b)} b_h \]

- \( \bar{N}_{i(b)} \): number density

\[ \bar{N}_{i(b)} = \int_0^\infty dz \frac{d^2V}{dz d\Omega} \int_0^\infty dM \frac{dn}{dM} S_{i(b)}(M, z) \]

- \( S_{i(b)} \): selection fn

Gray-body factor

Redshift binning

\[ S_{i(b)}(M, z) = (b) \Theta(z - z_i) \Theta(z_{i+1} - z) \]

Mass-observable relation

\[ \times \frac{1}{2} \left[ \text{erfc} \left( x(M_{(b)}; M) \right) - \text{erfc} \left( x(M_{(b+1)}; M) \right) \right] \]
Fisher matrix formalism

✓ Fisher matrix
\[ F_{\alpha \beta} = \sum_{\ell = \ell_{\text{min}}}^{\ell_{\text{max}}} \sum_{I,J} \frac{\partial C_I(\ell)}{\partial \theta^\alpha} \left[ \text{Cov}(C(\ell), C(\ell)) \right]^{-1} \frac{\partial C_J(\ell)}{\partial \theta^\beta} \]

✓ Covariance matrix
\[ \text{Cov} \left[ C_{i(bb')}(\ell), C_{j(\tilde{bb}')} (\ell') \right] = \frac{\delta^K_{i,j} \delta^K_{\ell \ell'}}{(2\ell + 1) \Delta \ell} \frac{f^{(bb')}_{\text{sky}}}{f^{(\tilde{bb}')}_{\text{sky}}} \left( f^{(bb')}_{\text{sky}} f^{(\tilde{bb}')}_{\text{sky}} \right) \times \left[ \hat{C}_{i(bb)}(\ell) \hat{C}_{i(\tilde{bb}')} (\ell) + \hat{C}_{i(bb)}(\ell) \hat{C}_{i(bb')} (\ell) \right], \]

✓ Estimated angular power spectrum

Cosmic variance
\[ \hat{C}_{i(bb')} (\ell) = C_{i(bb')} (\ell) + \frac{1}{N_i(b)} \delta^K_{bb'} \]

Shot noise
\[ \text{N} \sim 10^8 \text{ (SKA1, Euclid), } 10^9 \text{ (SKA2)} \]
Mass-observable relation

✓ Probability of assigning the observable mass $M_{\text{obs}}$ to the true mass $M$ : log-normal distribution [Lima+Hu(2004)]

$$p(M_{\text{obs}}|M) = \frac{1}{\sqrt{2\pi}\sigma_{\ln M}} \exp \left[ -x^2(M_{\text{obs}}; M) \right]$$

with

$$x(M_{\text{obs}}; M) = \frac{\ln M_{\text{obs}} - \ln M - \ln M_{\text{bias}}}{\sqrt{2}\sigma_{\ln M}}$$

Systematic errors

$$\ln M_{\text{bias}}(M, z) = \ln M_{b,0}$$

$$+ \sum_{i=1}^{3} q_{b,i} \left[ \ln \left( \frac{M}{M_{\text{piv}}} \right) \right]^i + \sum_{i=1}^{3} s_{b,i} z^i,$$

$$\sigma_{\ln M}(M, z) = \sigma_{\ln M,0}$$

$$+ \sum_{i=1}^{3} q_{\sigma_{\ln M},i} \left[ \ln \left( \frac{M}{M_{\text{piv}}} \right) \right]^i + \sum_{i=1}^{3} s_{\sigma_{\ln M},i} z^i.$$