

# Recent developments in *open Inflation*

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# Recent developments in *open Inflation*

*Inflation with  
quantum tunneling*

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Part 1

# **INTRODUCTION**

# Inflation

- Almost scale invariant spectrum
- Highly Gaussian fluctuations
- Spatially quite flat

[Planck2015]

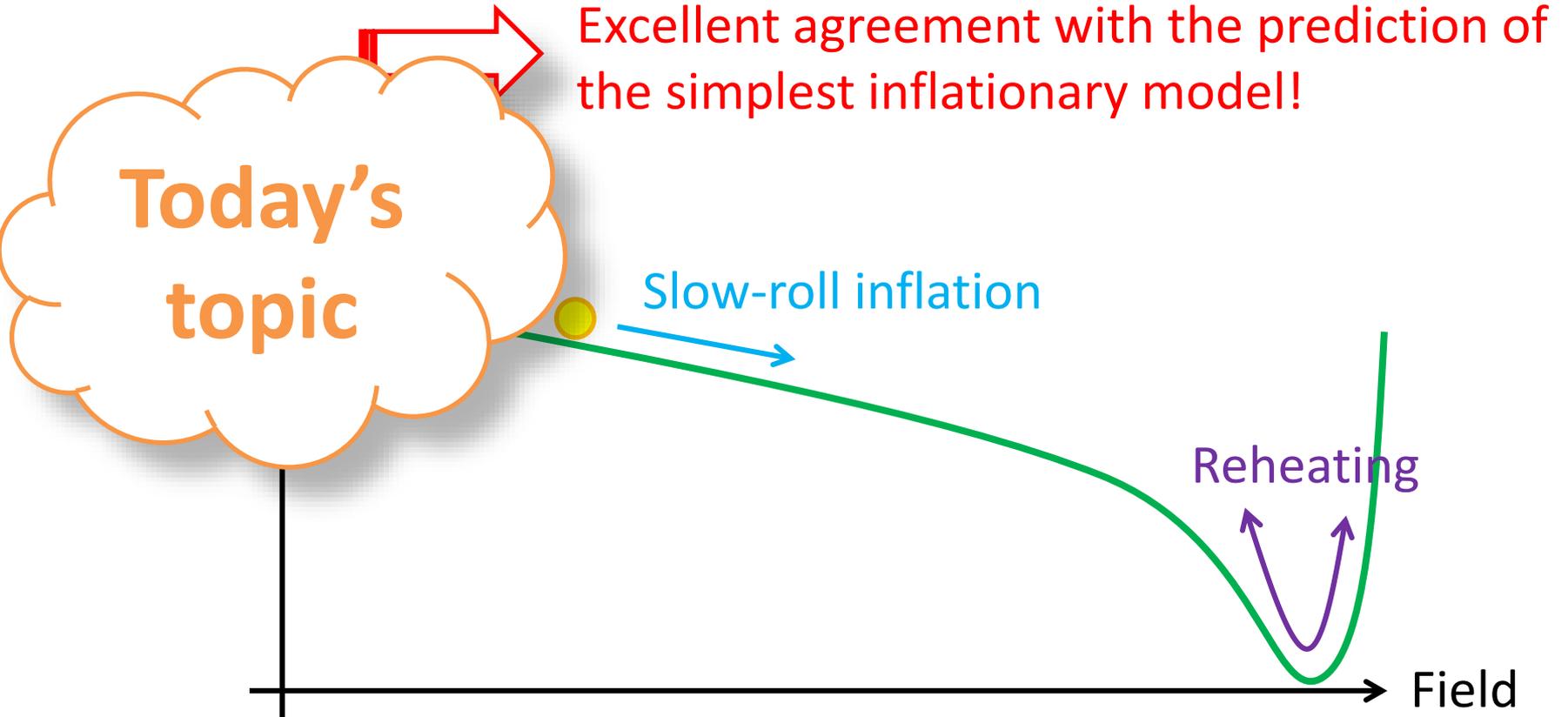
Excellent agreement with the prediction of the simplest inflationary model!

Today's  
topic

Slow-roll inflation

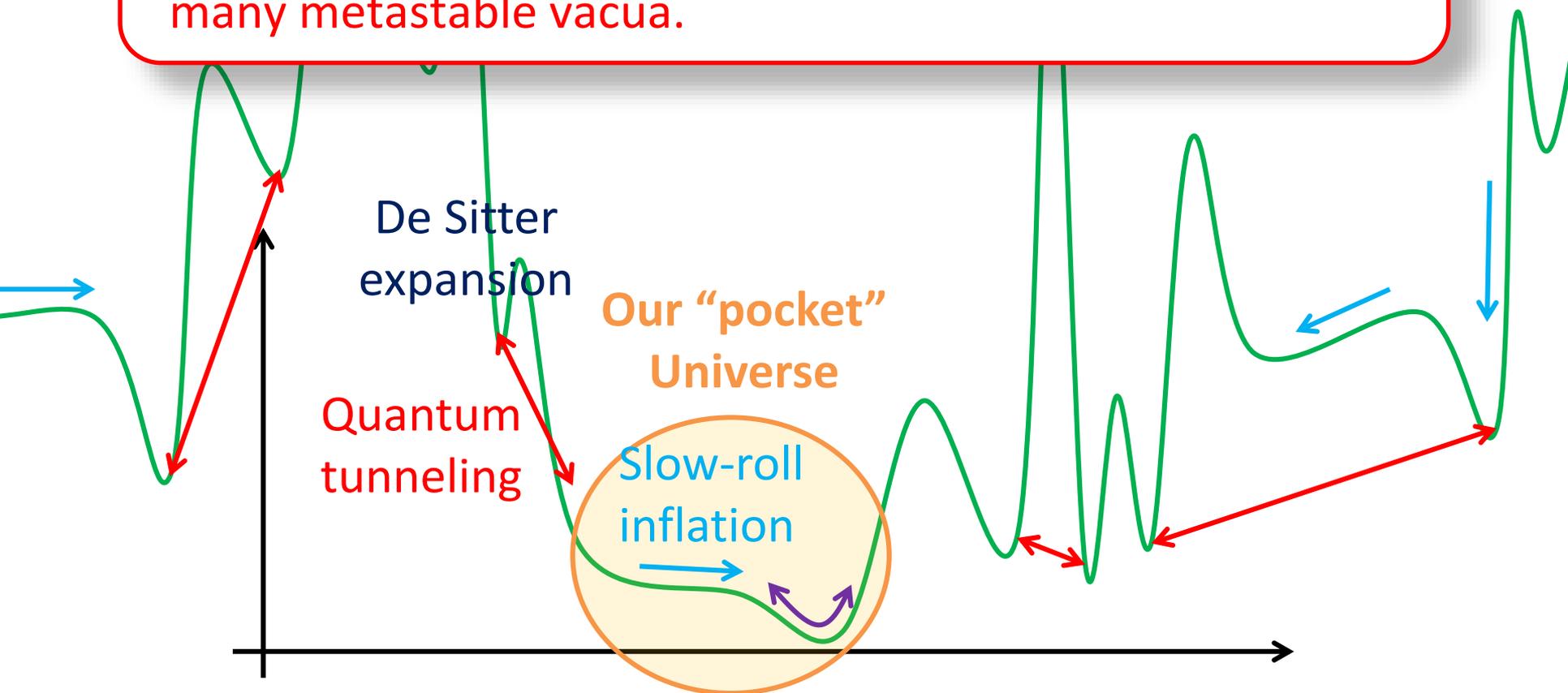
Reheating

Field

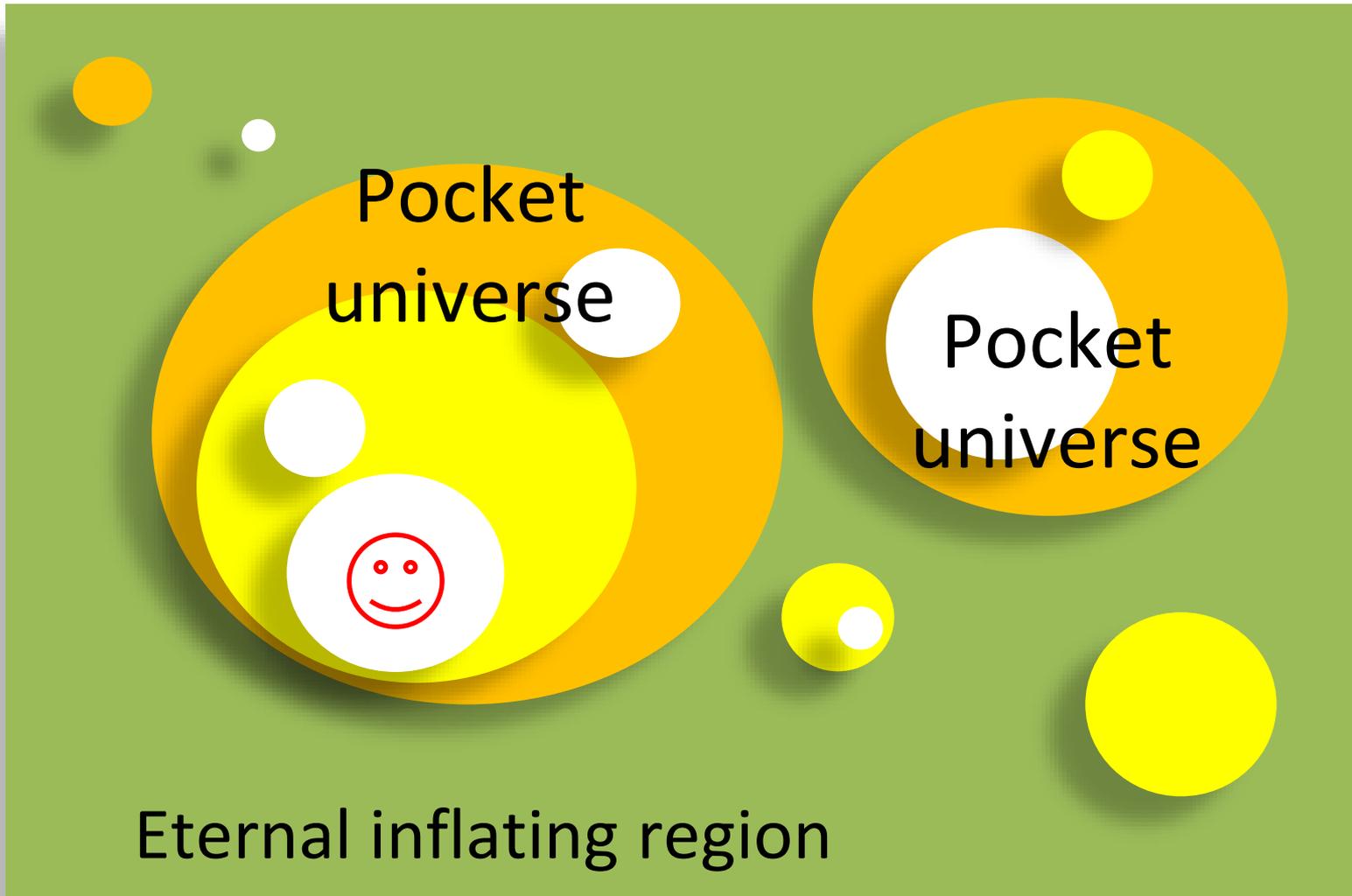


# Neighborhood we live in

It is quite possible that our part of the Universe appeared as a result of quantum tunneling after trapped in one of many metastable vacua.



# Whole universe



# String theorists consider...

$O(10^{500})$  metastable dS states with different properties depending on vev of scalar fields, compactifications,...



## “String landscape” picture

The inflationary multiverse becomes divided into an exponentially large number of different exponentially large “pocket universes” with different laws of low-energy physics operating in each of them.

*Is there any way to know what kind  
of neighborhood we live in ?*

*Revisit “Open Inflation” !*

# Plan

1. Introduction (finish)

2. Open inflation

3. Recent developments

3.1. Effect of rapid-rolling

3.2. CMB asymmetry

4. Future prospects

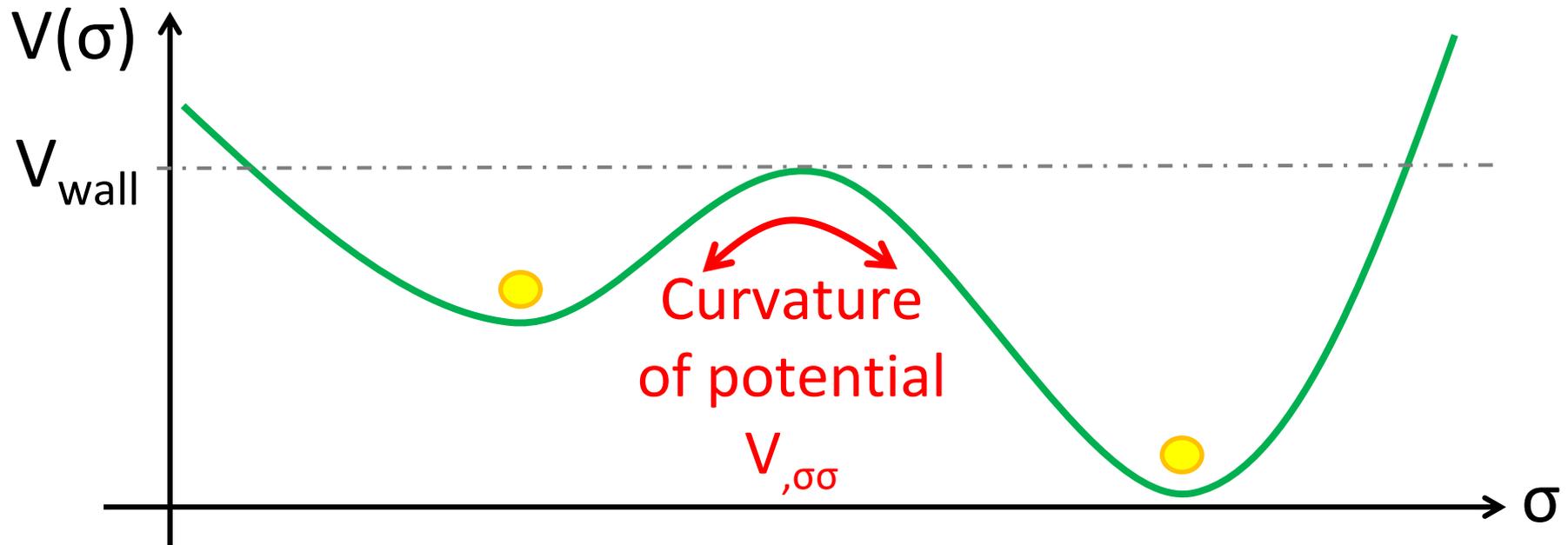
Part 2

# **OPEN INFLATION**

## **–BACKGROUND EVOLUTION–**

# Two possibilities

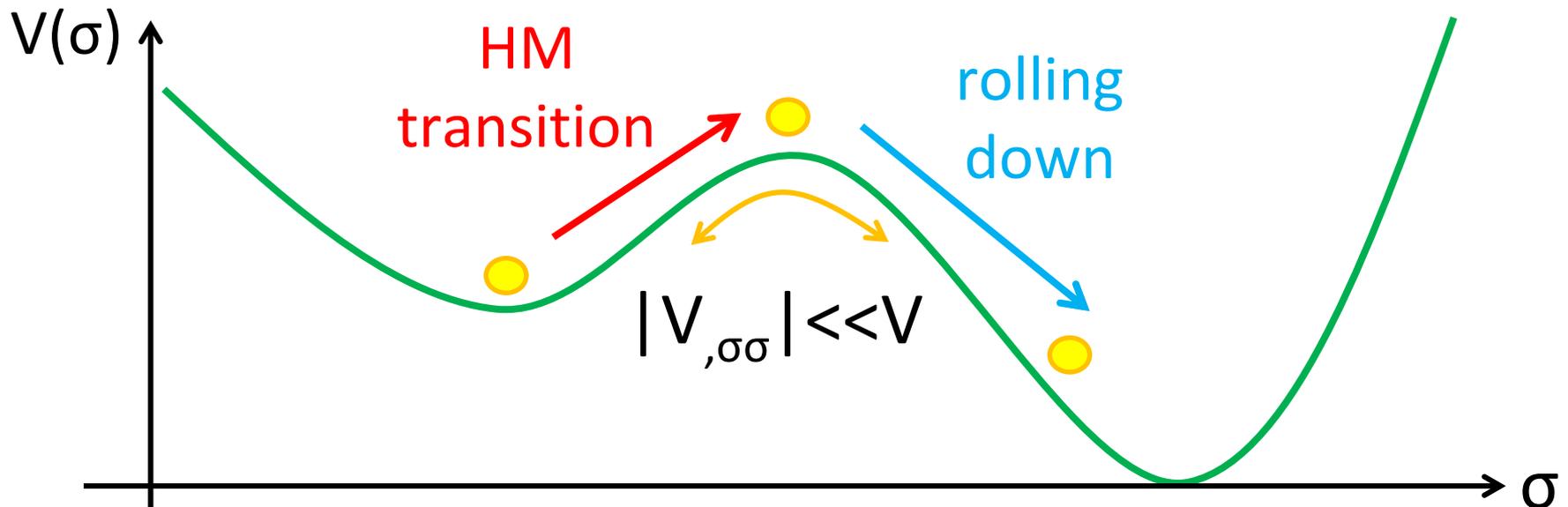
$M_{pl}=1$  unit



- $|V_{,\sigma\sigma}| \ll V_{wall} \rightarrow$  Hawking-Moss tunneling
- $|V_{,\sigma\sigma}| \gg V_{wall} \rightarrow$  Coleman-de Luccia tunneling

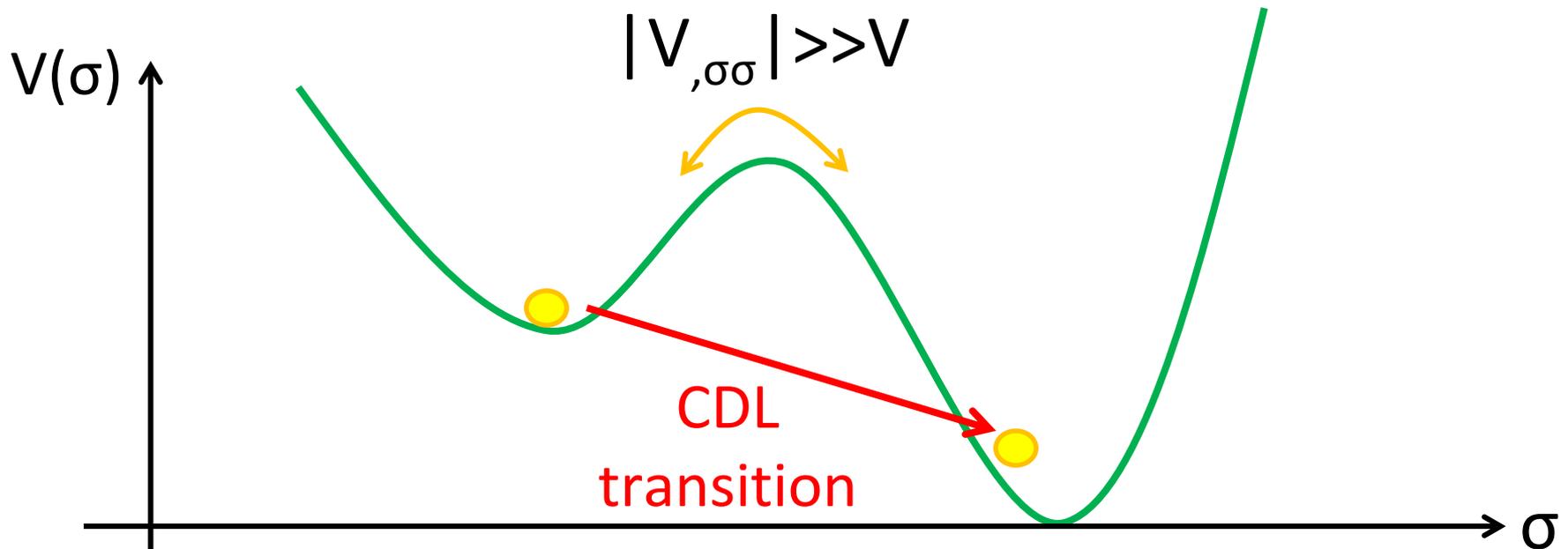
# Hawking-Moss (HM) tunneling

- Tunneling to a top of the barrier due to the quantum fluctuations
- ✓ Too large density perturbations unless  $e\text{-folds} \gg 60$  [Linde(1995)]
  - If the final transition is HM, we can not see any deviation from the standard prediction of slow-roll inflation.



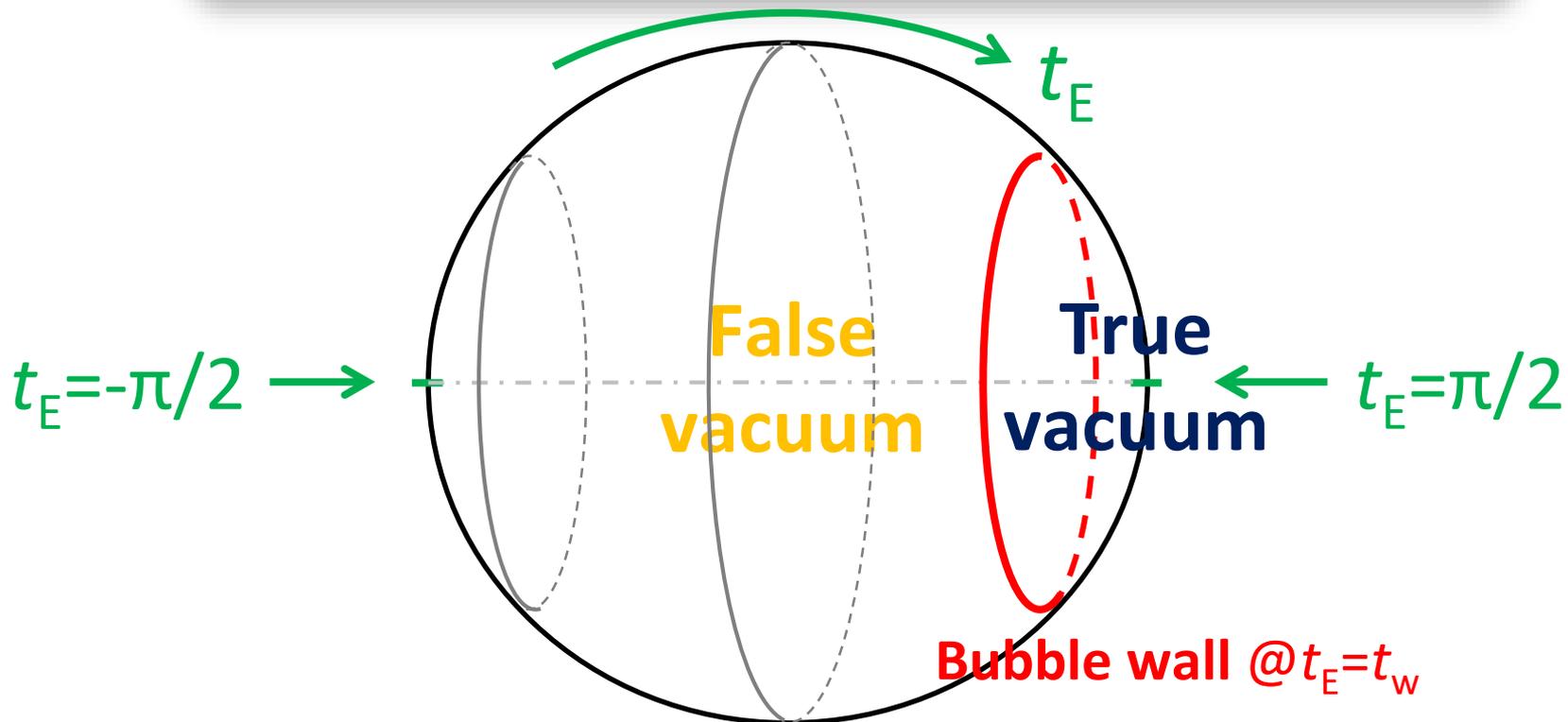
# Coleman-de Luccia (CDL) tunneling

- Tunneling mediated by an  $O(4)$ -symmetric solution of Euclidean Einstein-scalar equations.
- ✓ Reflecting the symmetry of a tunneling process, the region inside the bubble becomes an **open** FLRW universe!



# O(4) symmetric CDL bounce solution

$$ds^2 = dt_E^2 + \cos^2 t_E (dr_E^2 + \sin^2 r_E d\Omega^2)$$



- ✓ Note that an O(4)-sym solution is expected to be most likely to occur (at least in the case of vacuum decay without gravity).

# Background evolution in CDL

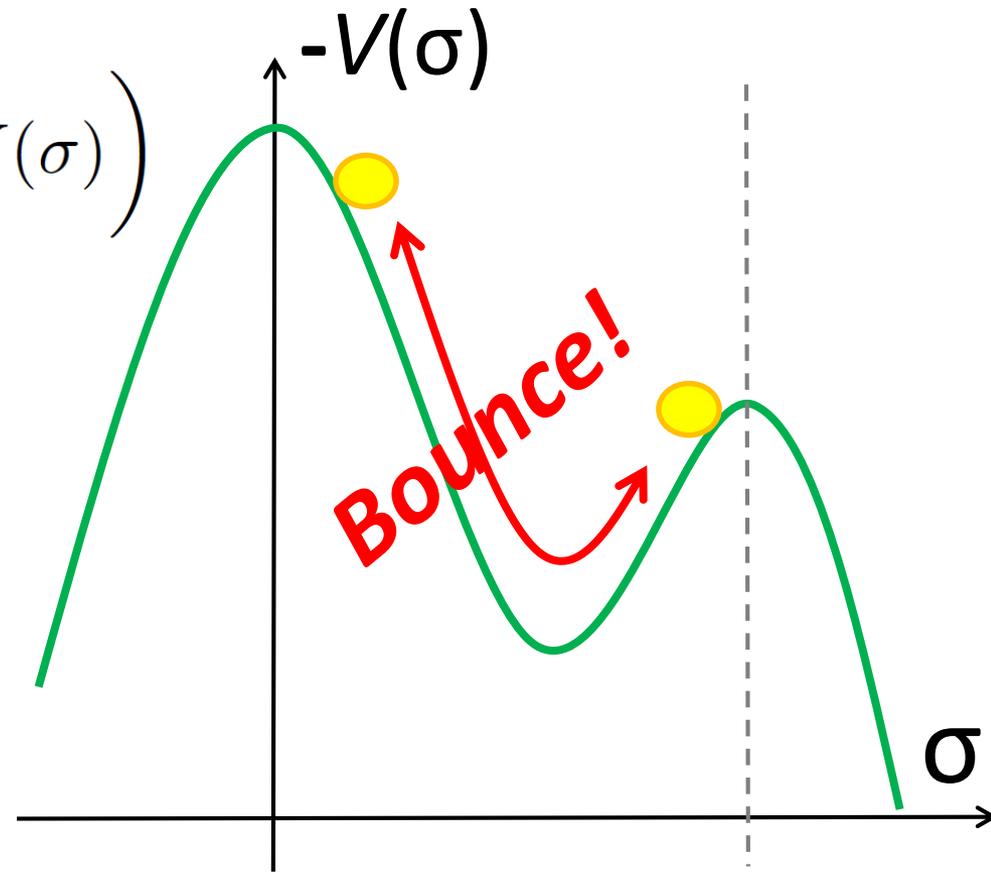
$$ds^2 = dt_E^2 + a^2(t_E) (dr_E^2 + \sin^2 r_E d\Omega^2)$$

## ✓ Basic equations

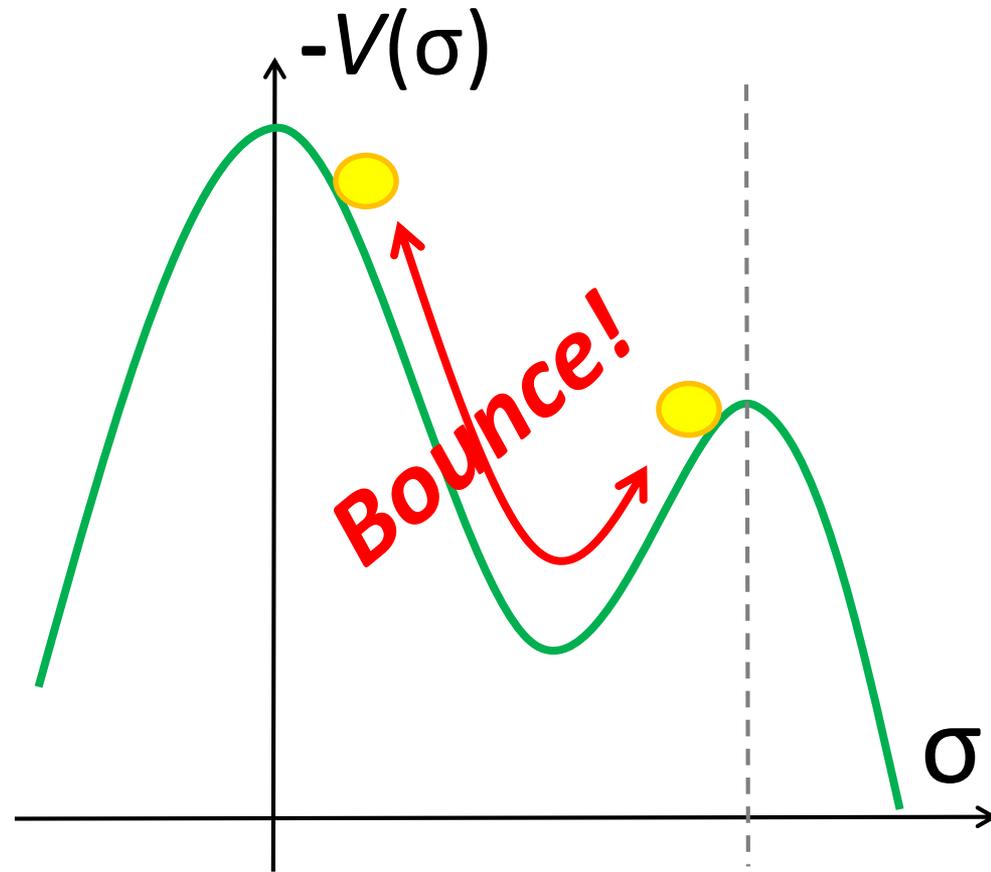
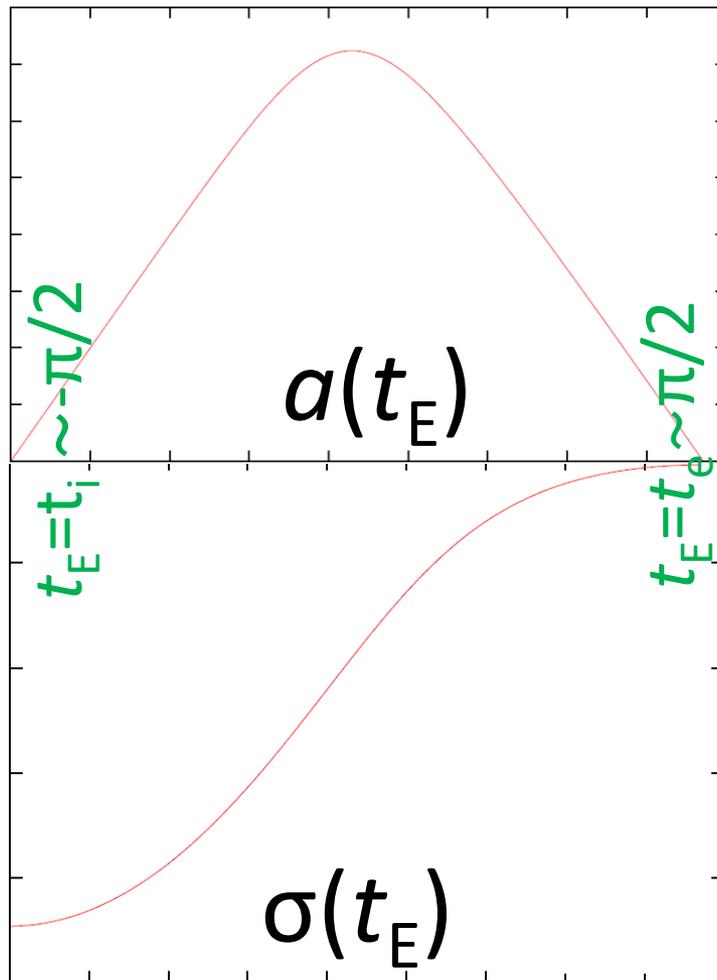
$$\left\{ \begin{array}{l} H^2 - \frac{1}{a_E^2} = \frac{1}{3} \left( \frac{1}{2} \dot{\sigma}^2 - V(\sigma) \right) \\ \dot{H} + \frac{1}{a_E^2} = -\frac{1}{2} \dot{\sigma}^2, \\ \ddot{\sigma} + 3H\dot{\sigma} - V' = 0. \end{array} \right.$$

## ✓ Boundary condition

$$\left\{ \begin{array}{l} \dot{\sigma}(t_i) = 0, \quad \dot{a}(t_i) = 1, \\ \dot{\sigma}(t_e) = 0, \quad \dot{a}(t_e) = -1. \end{array} \right.$$

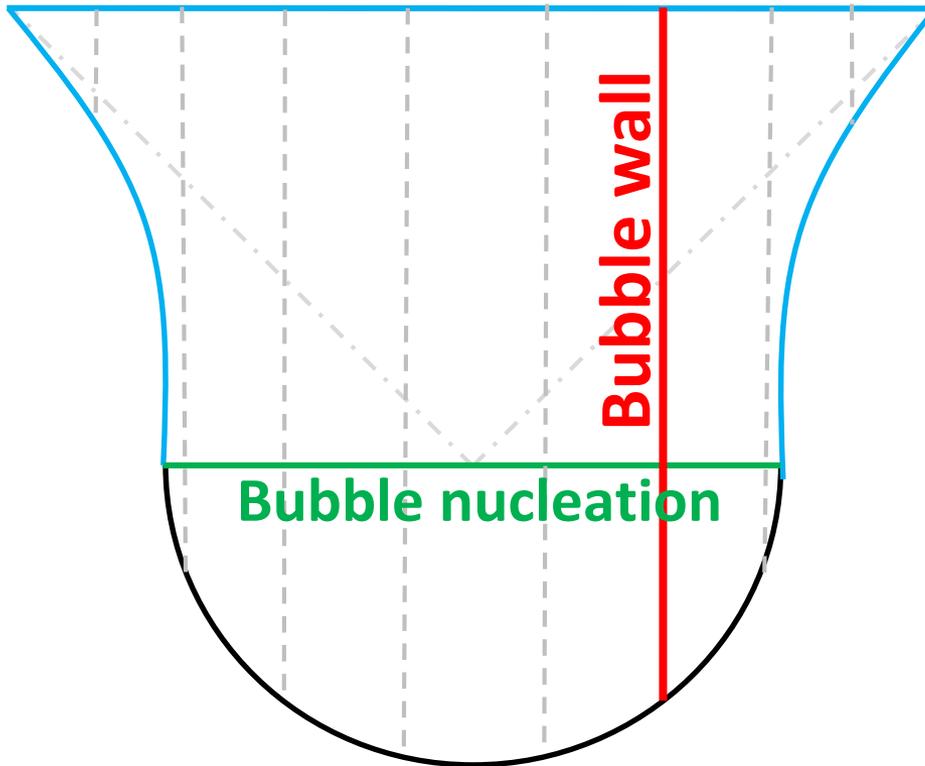


# Background evolution in CDL



# Geometry after CDL

are obtained by analytic continuation of an CDL solution from Euclidean to Lorentzian:



$$-T^2 + R^2 = 1/H^2$$

**Hyperbolic**



$$T_E^2 + R^2 = 1/H^2$$

**Spherical**

$$X^2+Y^2+Z^2+U^2+T_E^2=H^{-2}$$

$$T_E=\cos[t_E]\cos[r_E]$$

$$U=\sin[t_E]$$

$$Z=\cos[t_E]\sin[r_E]\cos[\theta]$$

$$X=\cos[t_E]\sin[r_E]\sin[\theta]\cos[\phi]$$

$$Y=\cos[t_E]\sin[r_E]\sin[\theta]\sin[\phi]$$



$$T_E \rightarrow iT$$

$$X^2+Y^2+Z^2+U^2-T^2=H^{-2}$$

$$T=\sinh[t_R]\cosh[r_R]$$

$$U=\cosh[t_R]$$

$$Z=\sinh[t_R]\sinh[r_R]\cos[\theta]$$

$$X=\sinh[t_R]\sinh[r_R]\sin[\theta]\cos[\phi]$$

$$Y=\sinh[t_R]\sinh[r_R]\sin[\theta]\sin[\phi]$$

$$\begin{cases} t_E = -i t_R + \pi/2 = t_C \\ r_E = -i r_R = -i r_C + \pi/2 \end{cases}$$

$$T=\sinh[r_C]\cos[t_C]$$

$$U=\sin[t_C]$$

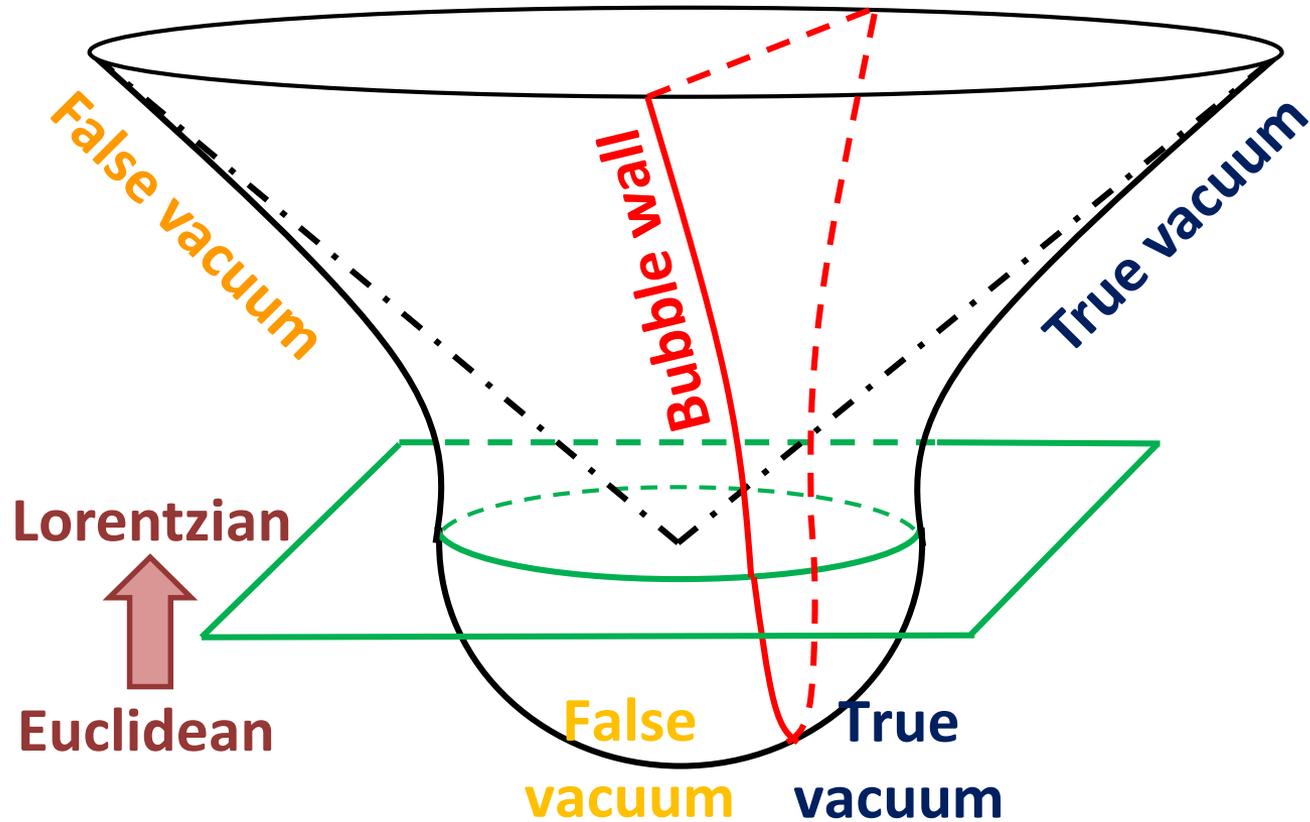
$$Z=\cosh[r_C]\cos[t_C]\cos[\theta]$$

$$X=\cosh[r_C]\cos[t_C]\sin[\theta]\cos[\phi]$$

$$Y=\cosh[r_C]\cos[t_C]\sin[\theta]\sin[\phi]$$

# Geometry after CDL

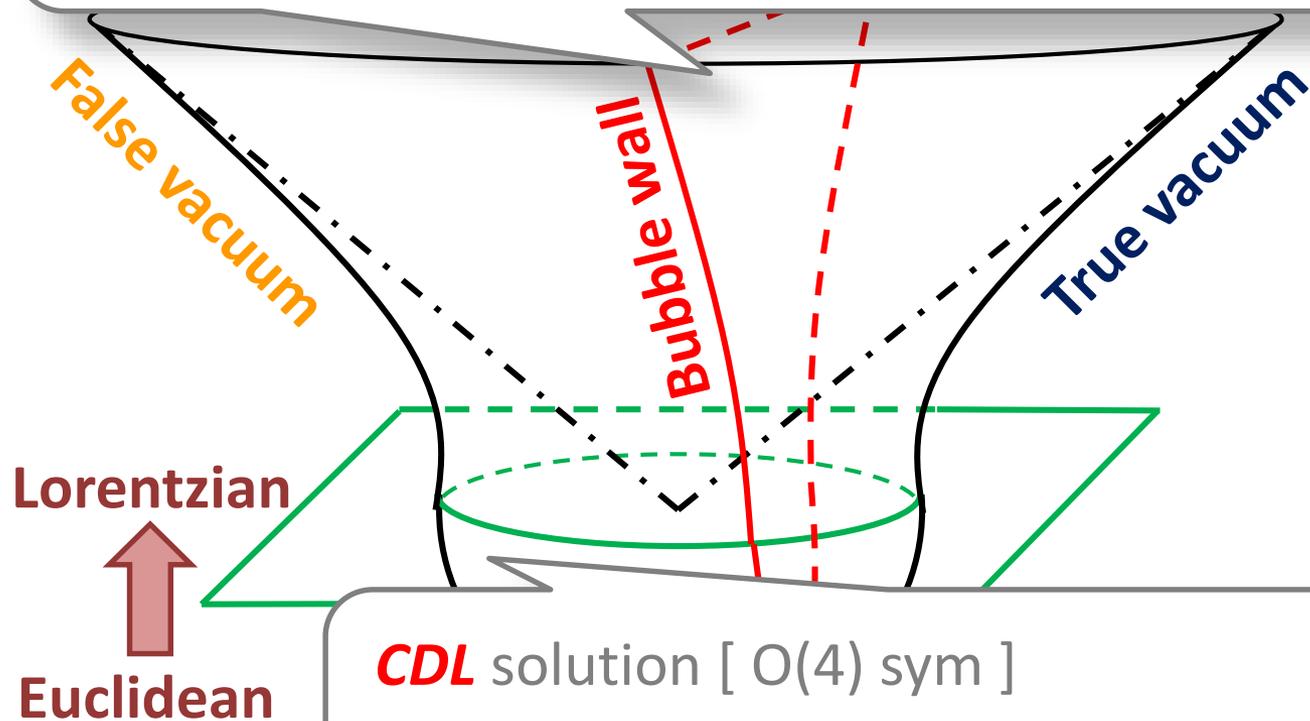
are obtained by analytic continuation of an CDL solution from Euclidean to Lorentzian:



# Geometry after CDL

an **Open**-slice of de Sitter [  $O(3,1)$  sym ]  
from

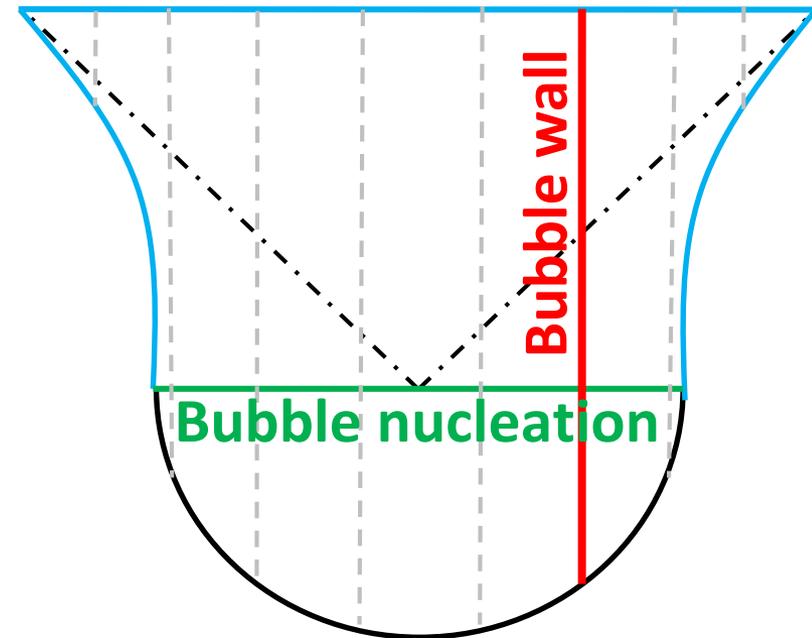
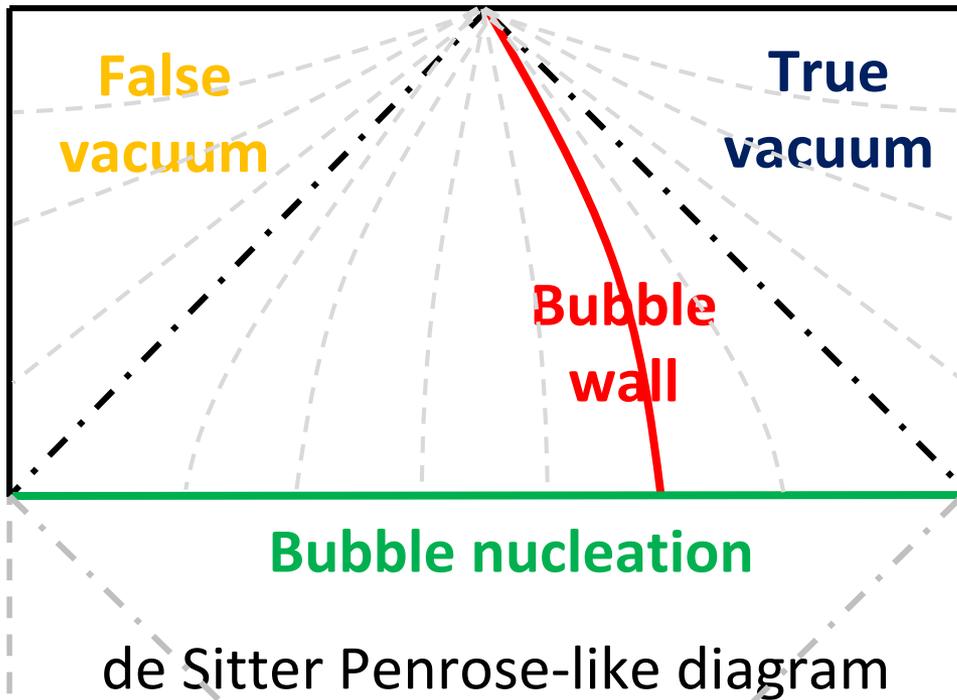
$$ds^2 = -dt_R^2 + \sinh^2 t_R (dr_R^2 + \sinh^2 r_R d\Omega^2)$$



**CDL** solution [  $O(4)$  sym ]

$$ds^2 = dt_E^2 + \cos^2 t_E (dr_E^2 + \sin^2 r_E d\Omega^2)$$

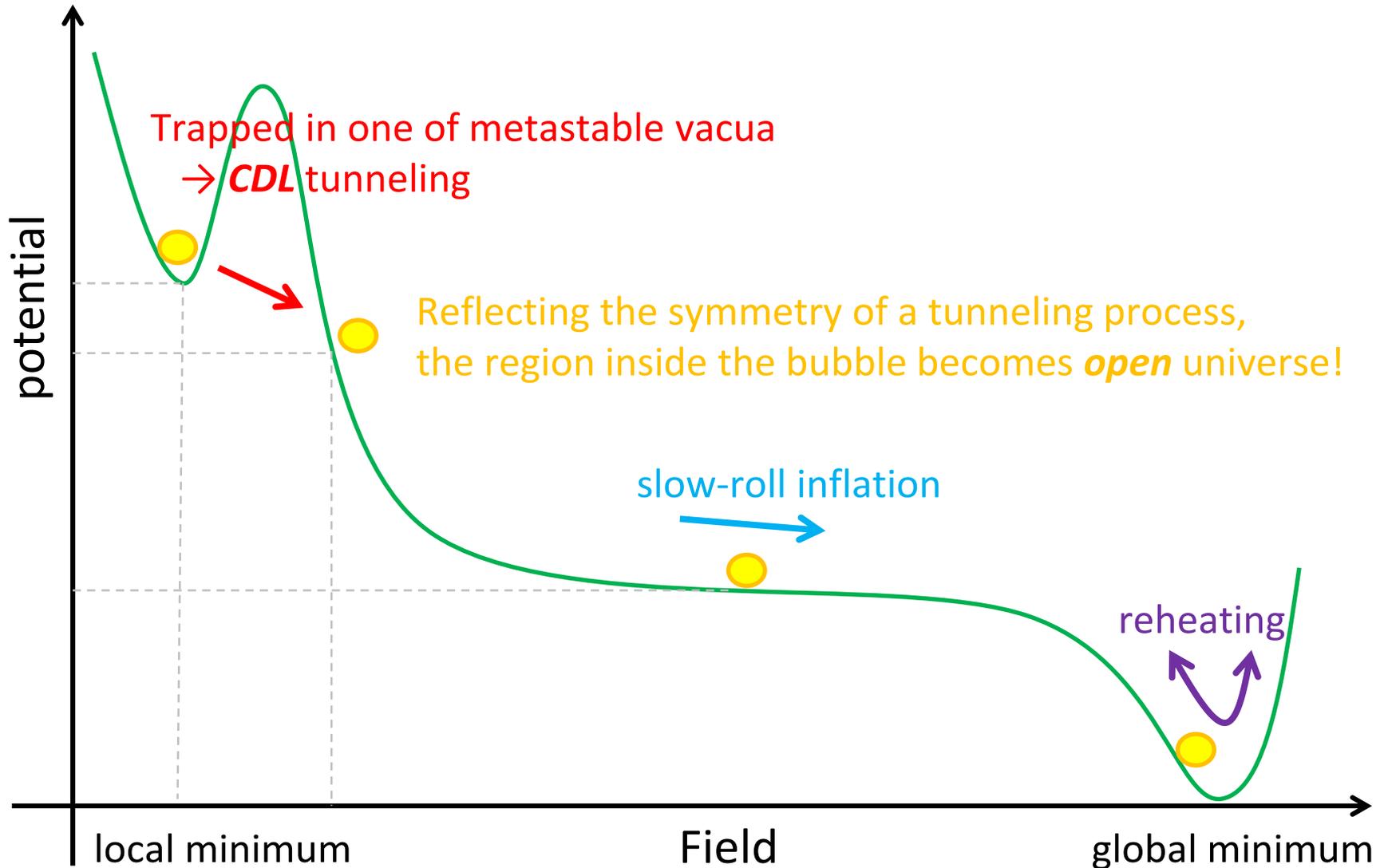
# Geometry after CDL



*Open*-slice of de Sitter [  $O(3,1)$  sym ]

$$ds^2 = -dt_R^2 + \sinh^2 t_R (dr_R^2 + \sinh^2 r_R d\Omega^2)$$

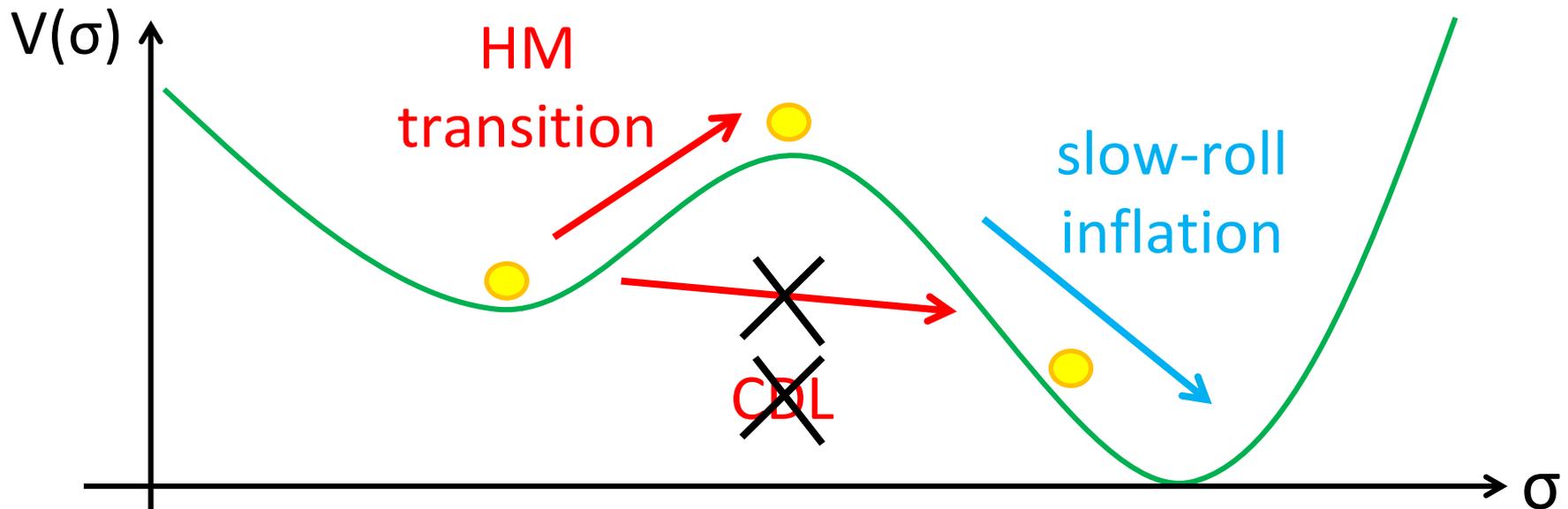
# Open Inflation



# Ex. 1 : Simplest polynomial potential

$$V(\sigma) = \frac{m^2}{2}\sigma^2 - \frac{\delta}{3}\sigma^3 + \frac{\lambda}{4}\sigma^4$$

- $|V_{,\sigma\sigma}| < 4H^2$  : **HM tunneling** [Linde (1999)]
- The condition for CDL and subsequent slow-roll inflation are not easily satisfied at the same time !

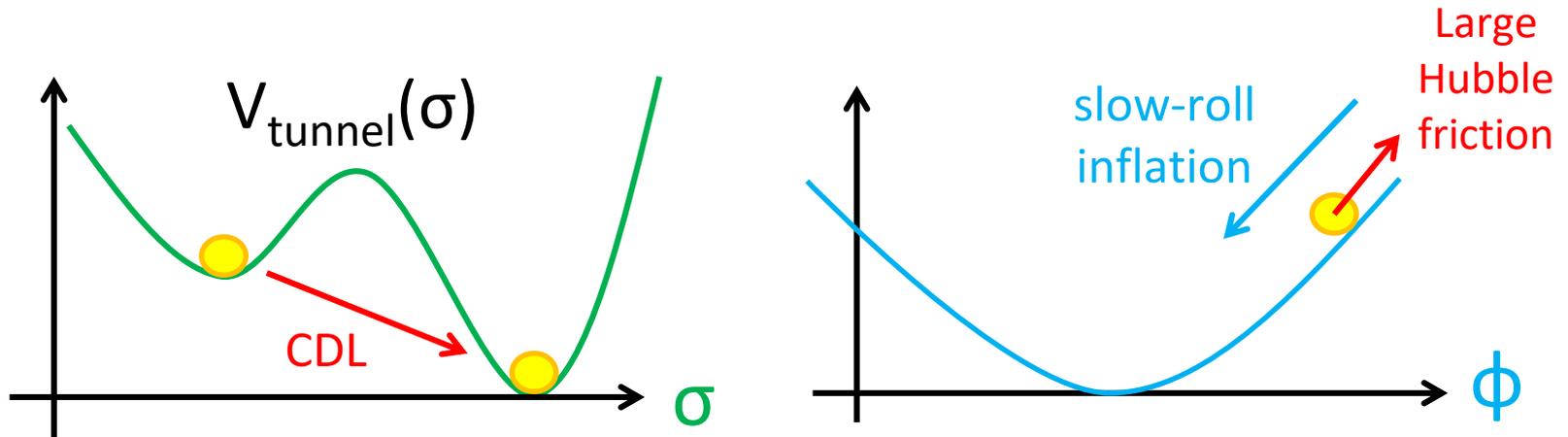


# Ex. 2 : Simple two-field model

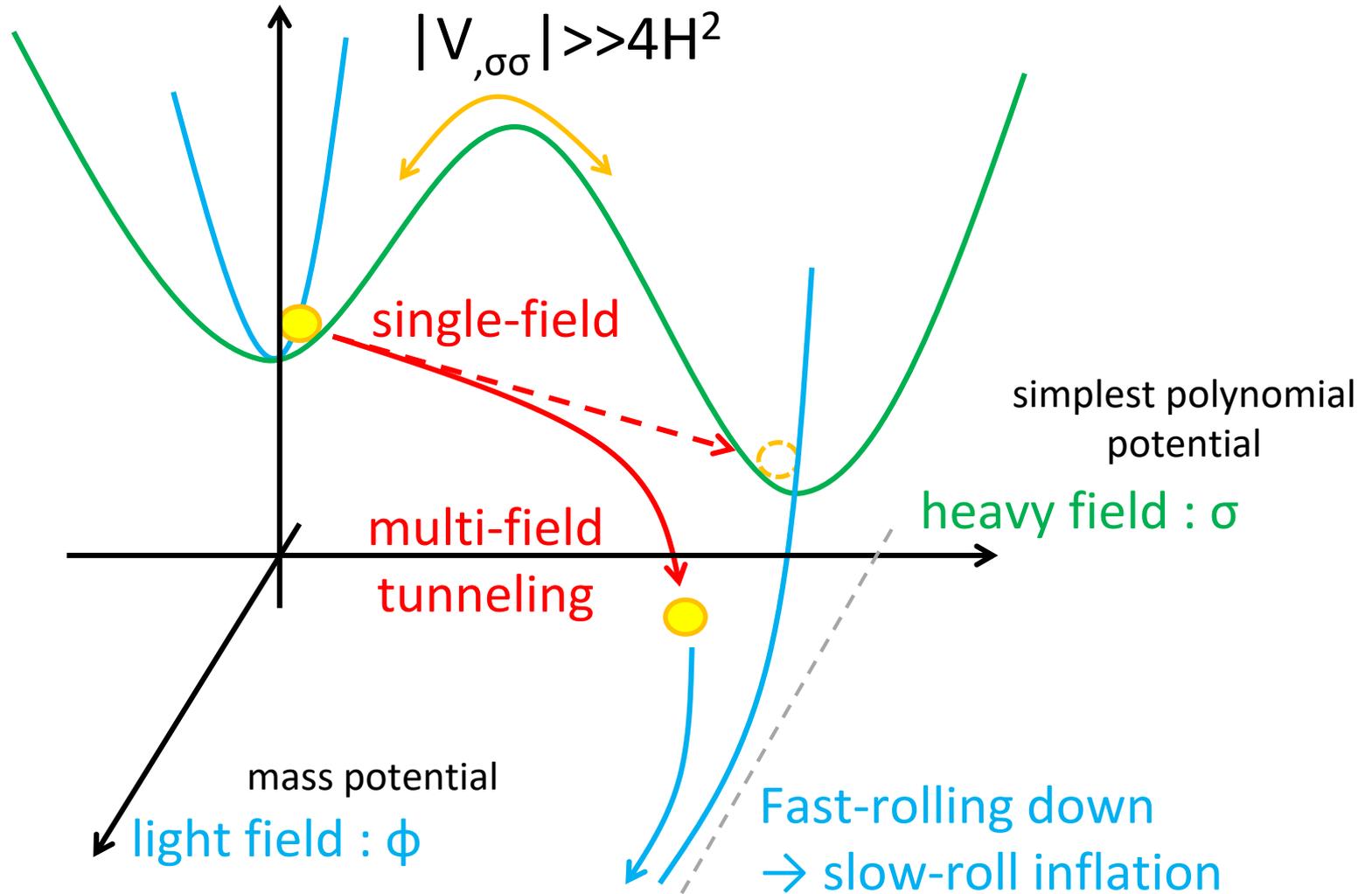
- Naturally/easily realized in the landscape
  - $\sigma$  : heavy field  $\rightarrow$  false vacuum decay
  - $\phi$  : light field  $\rightarrow$  starts rolling after FV decay

$$V(\sigma, \phi) = V_{\text{tunnel}}(\sigma) + m^2 \phi^2 / 2$$

- ✓ Too large perturbations from supercurvature mode of  $\phi$  unless e-folds  $\gg 60$  [Sasaki+Tanaka(1996)]



# Ex. 3 : Multi-field tunneling and inflation



# Ex. 3 : Tunneling rate $\sim \exp(-B)$

Mass of light-field  $\phi$  at TV

**multi-CDL** **effective single-CDL** **HM** **(multi) - (single)**

$m_\phi [m_{\text{pl}}]$	$\Delta\sigma [m_{\text{pl}}]$	$\Delta\phi [m_{\text{pl}}]$	$B$	$B_0$	$B_{\text{HM}}$	$\Delta B = B - B_0$
$10^{-6}$	1.91	$2.20 \times 10^{-10}$	12109.11	12109.11	12679.69	$ \Delta B  < 0.01$
$10^{-4}$	1.91	$2.20 \times 10^{-6}$	12108.10	12108.10	12678.65	$ \Delta B  < 0.01$
$10^{-3}$	1.91	$2.19 \times 10^{-4}$	12008.71	12008.71	12576.67	$ \Delta B  < 0.01$
$5 \times 10^{-3}$	1.90	$5.34 \times 10^{-3}$	9975.05	9975.07	10484.43	-0.02
$10^{-2}$	1.87	$1.97 \times 10^{-2}$	6322.66	6322.85	6691.28	-0.19
$2 \times 10^{-2}$	1.73	$6.00 \times 10^{-2}$	2188.07	2189.41	2305.67	-1.35
$3 \times 10^{-2}$	1.38	$8.73 \times 10^{-2}$	849.20	852.07	868.55	-2.87
$4 \times 10^{-2}$	0.49	$4.58 \times 10^{-2}$	372.15	376.39	372.28	-4.25

Multi-field dynamics tends to increase the tunneling-rate (?).

Part 2

# **OPEN INFLATION –FLUCTUATIONS–**

# Quantization

## Step 1

We need to find the reduced action that contains only the physical degree of freedom.

Ex) Simple scalar field

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{1}{2} m^2(\eta) \sigma^2 \right)$$

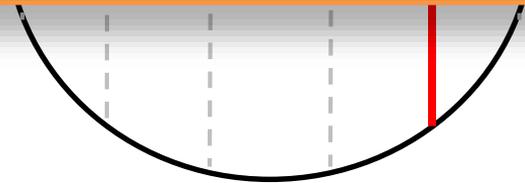
# Quantization

## Step 2

We need to find a complete set of functions which obey the field equation and which **are regular on the lower hemisphere**.

A set of all modes which can be Klein-Gordon normalized on a **Cauchy surface**

$$(\sigma_N, \sigma_M)_{\text{KG}} = -i \int_{\Sigma} d\Sigma_{\mu} g^{\mu\nu} \left\{ \sigma_N \partial_{\nu} \bar{\sigma}_M - (\partial_{\nu} \sigma_N) \bar{\sigma}_M \right\} = \delta_{NM}.$$



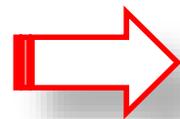
# Quantization

## Step 3

We promote the physical degree of freedom to operator and expand it by mode functions  $\{\sigma_N, \bar{\sigma}_N\}$  as

$$\hat{\sigma} = \sum_{\mathcal{N}} \left[ \hat{a}_{\mathcal{N}} \sigma_{\mathcal{N}} + \hat{a}_{\mathcal{N}}^{\dagger} \bar{\sigma}_{\mathcal{N}} \right]$$

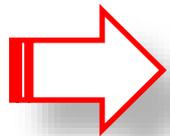
$$[\hat{a}_{\mathcal{N}}, \hat{a}_{\mathcal{M}}^{\dagger}] = \delta_{\mathcal{N}\mathcal{M}}$$



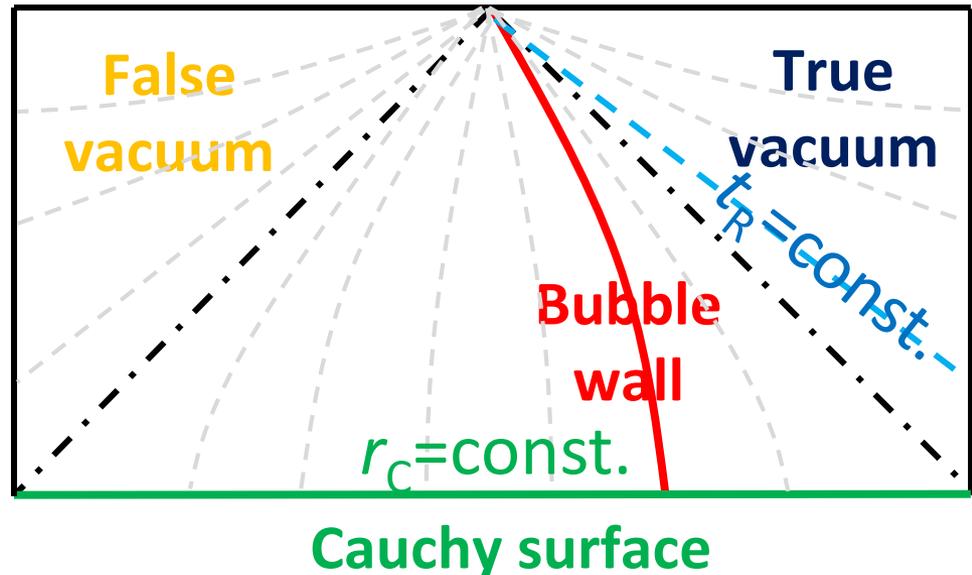
Mode function  $\sigma_N$  plays the role of positive frequency functions!

# Difficulty in quantization

The surfaces which respect the maximal sym. ( $t_R = \text{const.}$ ) are not the Cauchy surface of the whole spacetime.



We need to work in the center region ( $r_C = \text{const.}$ ), where the background configuration is spatially inhomogeneous.



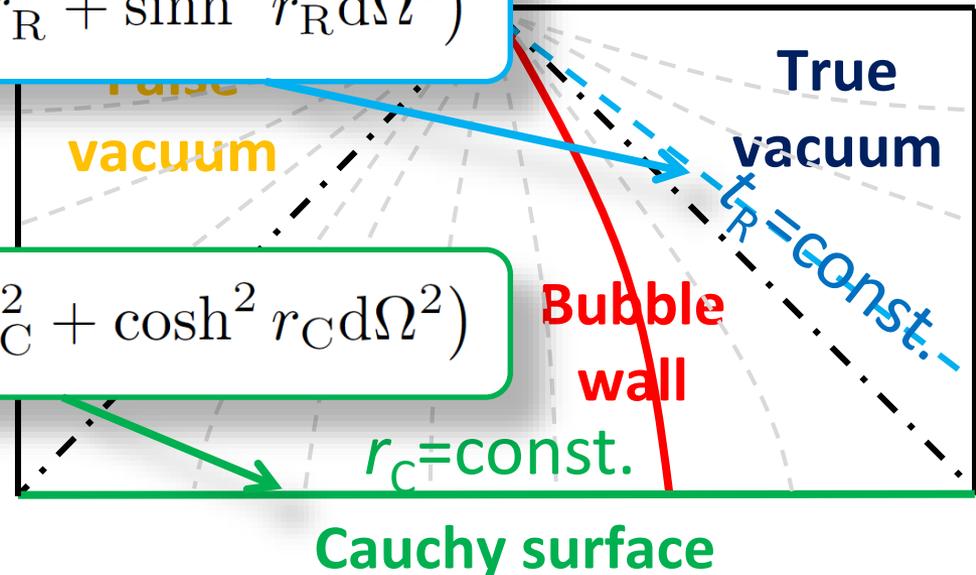
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$$ds^2 = -dt_R^2 + \sinh^2 t_R (dr_R^2 + \sinh^2 r_R d\Omega^2)$$

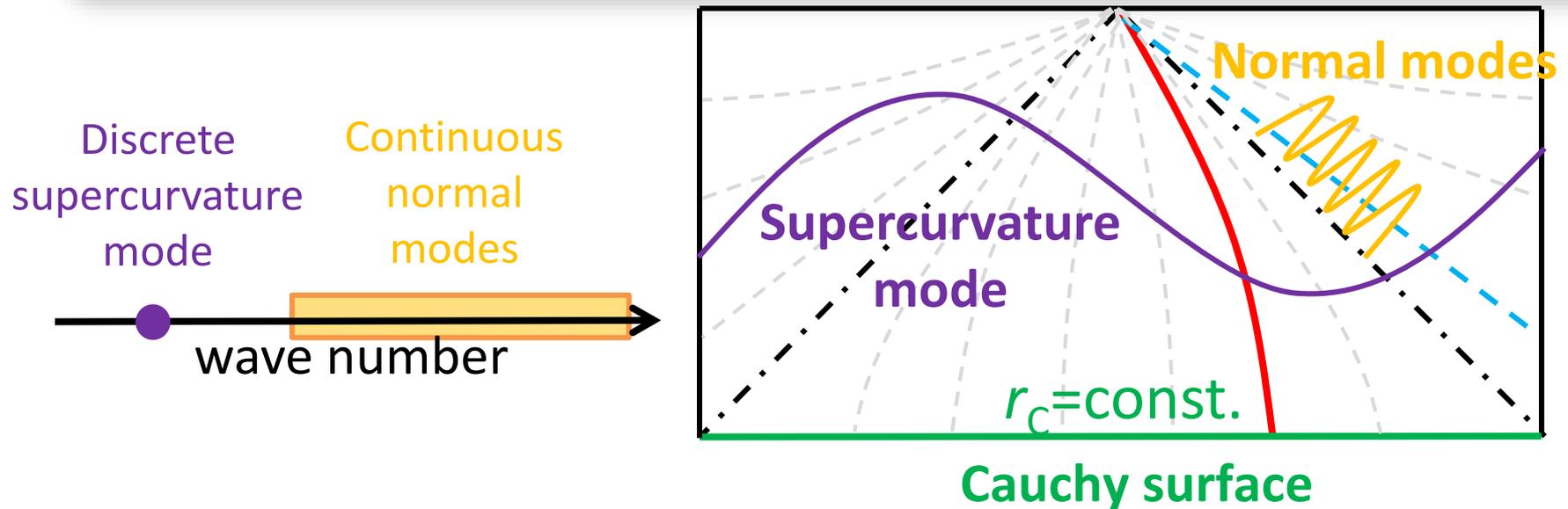
$$ds^2 = dt_C^2 + \cos^2 t_C (-dr_C^2 + \cosh^2 r_C d\Omega^2)$$



# (Discrete) supercurvature mode

There could appear a set of modes which have finite KG norms on Cauchy surfaces, but which *cannot be quantized on the open chart* because of the divergent KG norms on the open chart.

$$(\sigma_{\mathcal{N}}, \sigma_{\mathcal{M}})_{\text{KG}} = -i \int_{-\infty}^{\infty} d\eta_{\mathcal{C}} \int d\Omega a^2(\eta_{\mathcal{C}}) \left\{ \sigma_{\mathcal{N}} \partial_{r_{\mathcal{C}}} \bar{\sigma}_{\mathcal{M}} - (\partial_{r_{\mathcal{C}}} \sigma_{\mathcal{N}}) \bar{\sigma}_{\mathcal{M}} \right\} = \delta_{\mathcal{N}\mathcal{M}}$$



# (Discrete) supercurvature mode

There could appear a set of modes which have finite KG norms on Cauchy surfaces, but which *cannot be quantized on the open chart* because of the divergent KG norms on the open chart.

➤ Canonical scalar field →  $\left\{ \begin{array}{l} \times \text{ (for } m > 3H/2) \\ \circ \text{ (for } m \leq 3H/2) \end{array} \right.$   
[Sasaki+Tanaka+Yamamoto (1995)]

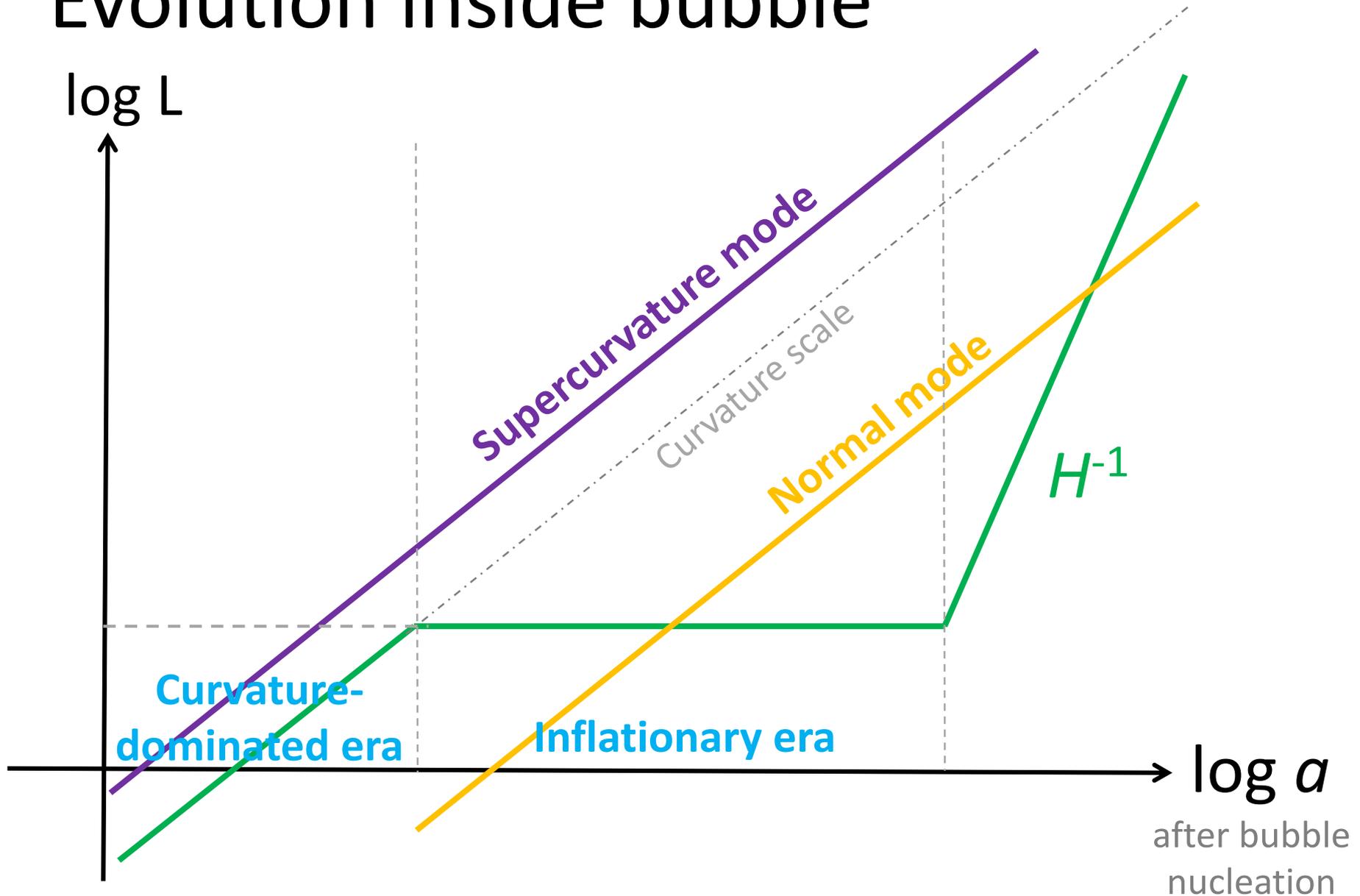
➤ Tensor perturbations →  $\times$   
[Tanaka+Sasaki (1997)]

$$k_{sc}^2 \simeq (2m^2/3H^2) k_{curv}^2$$

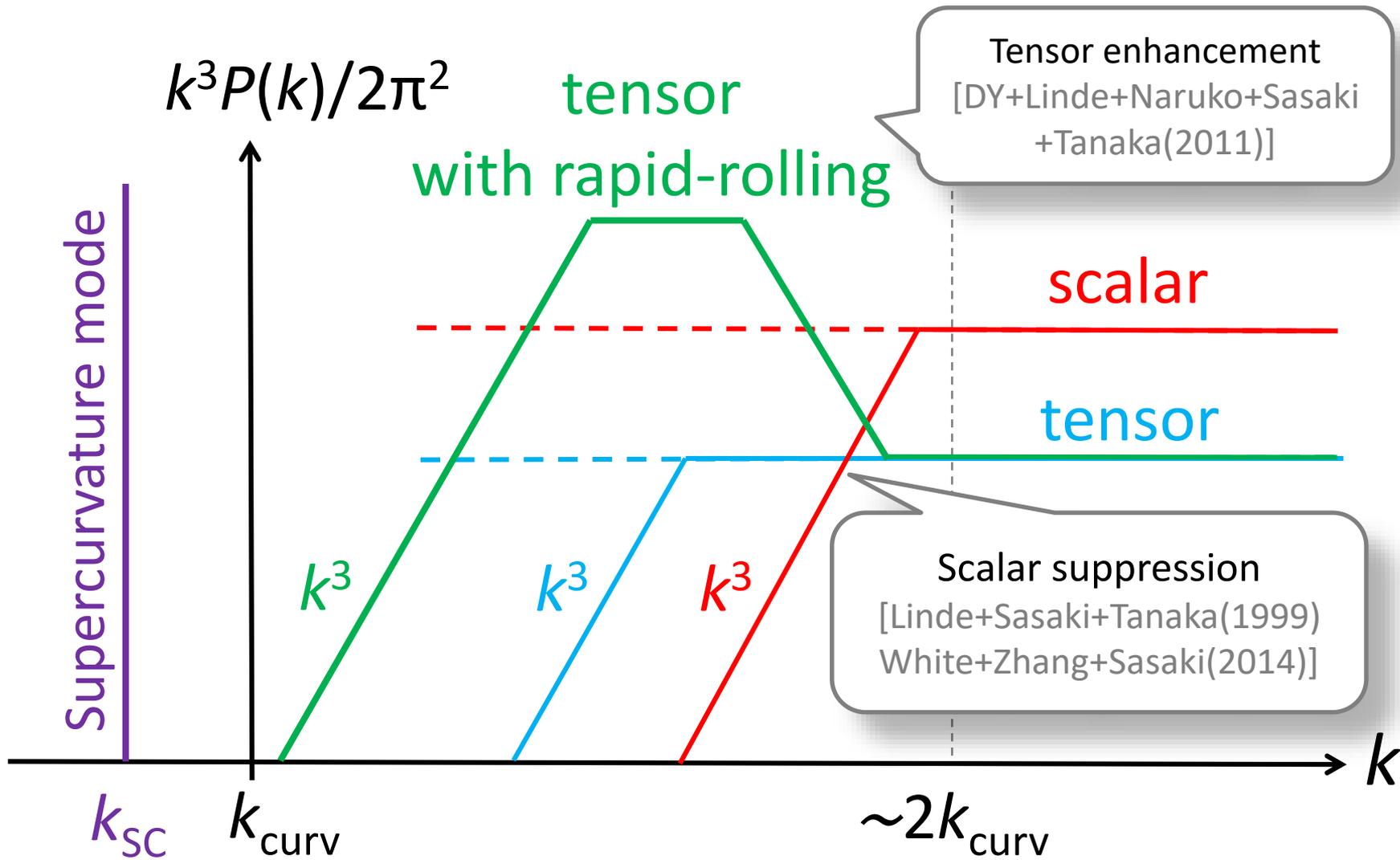
➤ U(1) gauge field →  $\times$   
[DY+Fujita+Mukohyama, 1402.2784]

➤ Metric pert.+scalar field →  $?$  (model-depend)  
[Garriga+Montes+Sasaki+Tanaka(1999)]

# Evolution inside bubble



# Power spectrum for scalar/tensor



(若干の適当さを含みます)

Part 3

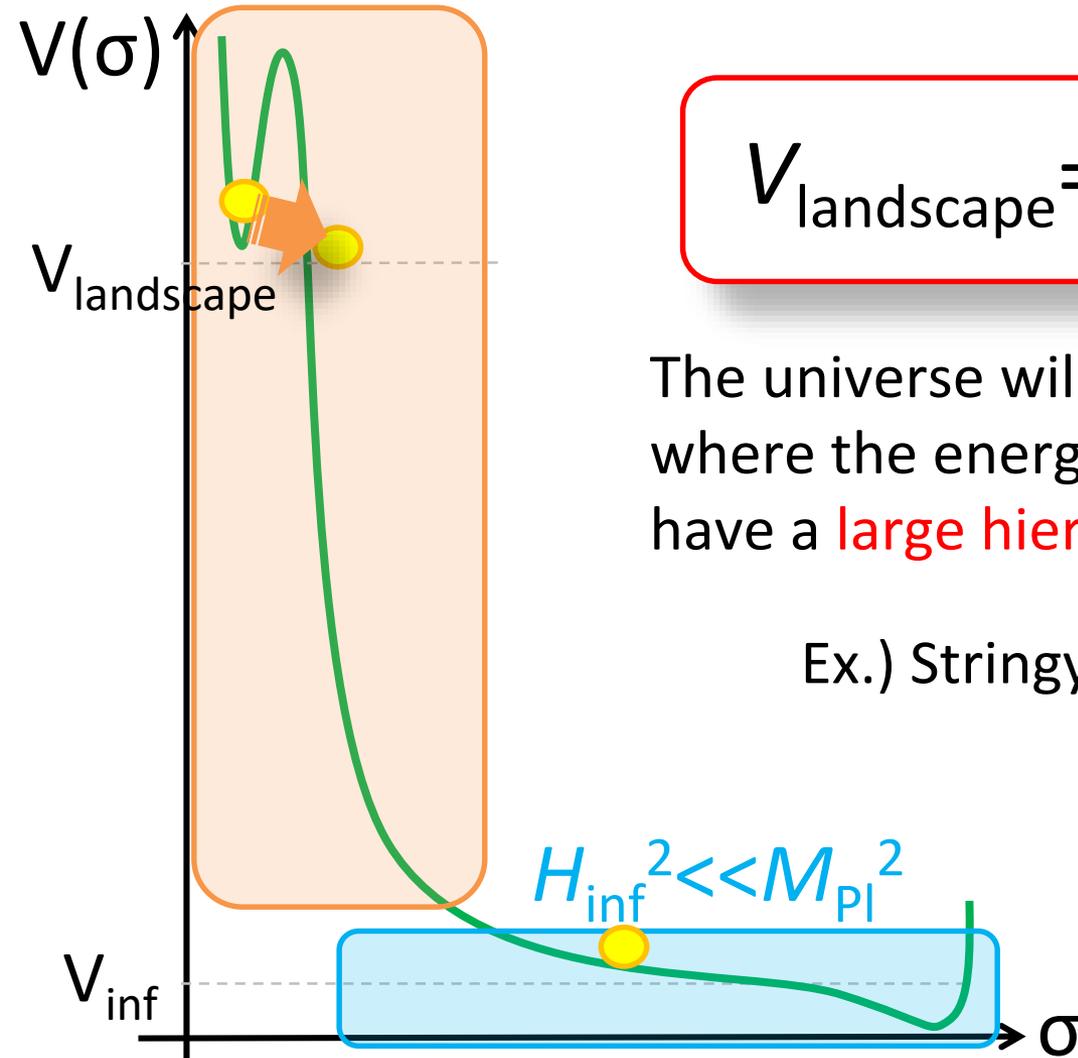
# **RECENT DEVELOPMENTS 1**

## **—SIGNALS FROM STRINGY INFLATION—**

# Classification for open inflation

Inflation driver ( $m < H$ )	Tunneling driver ( $m > H$ )	Curvature pert. $\zeta$	Supercurvature mode?	note
$\sigma$	$\sigma$	$\sigma$	$\sigma \rightarrow \text{O}/\times$	<ul style="list-style-type: none"> <li>▪ Linde(1998)</li> <li>▪ Linde+Sasaki+Tanaka(1999)</li> <li>▪ Garriga+Montes+Sasaki+Tanaka(1999)</li> <li>▪ DY+(2011)</li> </ul>
$\sigma$	$\rho$	$\sigma$	$\sigma \rightarrow \text{O}$	<ul style="list-style-type: none"> <li>▪ Linde+Mezhlumian(1995)</li> <li>▪ Sasaki+Tanaka(1996)</li> </ul>
$\varphi$	$\rho$	$\sigma$	$\sigma \rightarrow \text{O}$	<ul style="list-style-type: none"> <li>[▪ tunneling sol : Sugimura+DY+Sasaki(2011)]</li> </ul>
$\varphi$	$\rho$	$\chi$	$\sigma \rightarrow \text{O}$	<ul style="list-style-type: none"> <li>▪ Kanno+Sasaki+Tanaka(2013)</li> <li>▪ Brnes+Domenech+Sasaki+Takahashi(2016)</li> <li>[▪ Sugimura+DY+Sasaki(2012)]</li> </ul>

# Requirements for a single-field stringy model



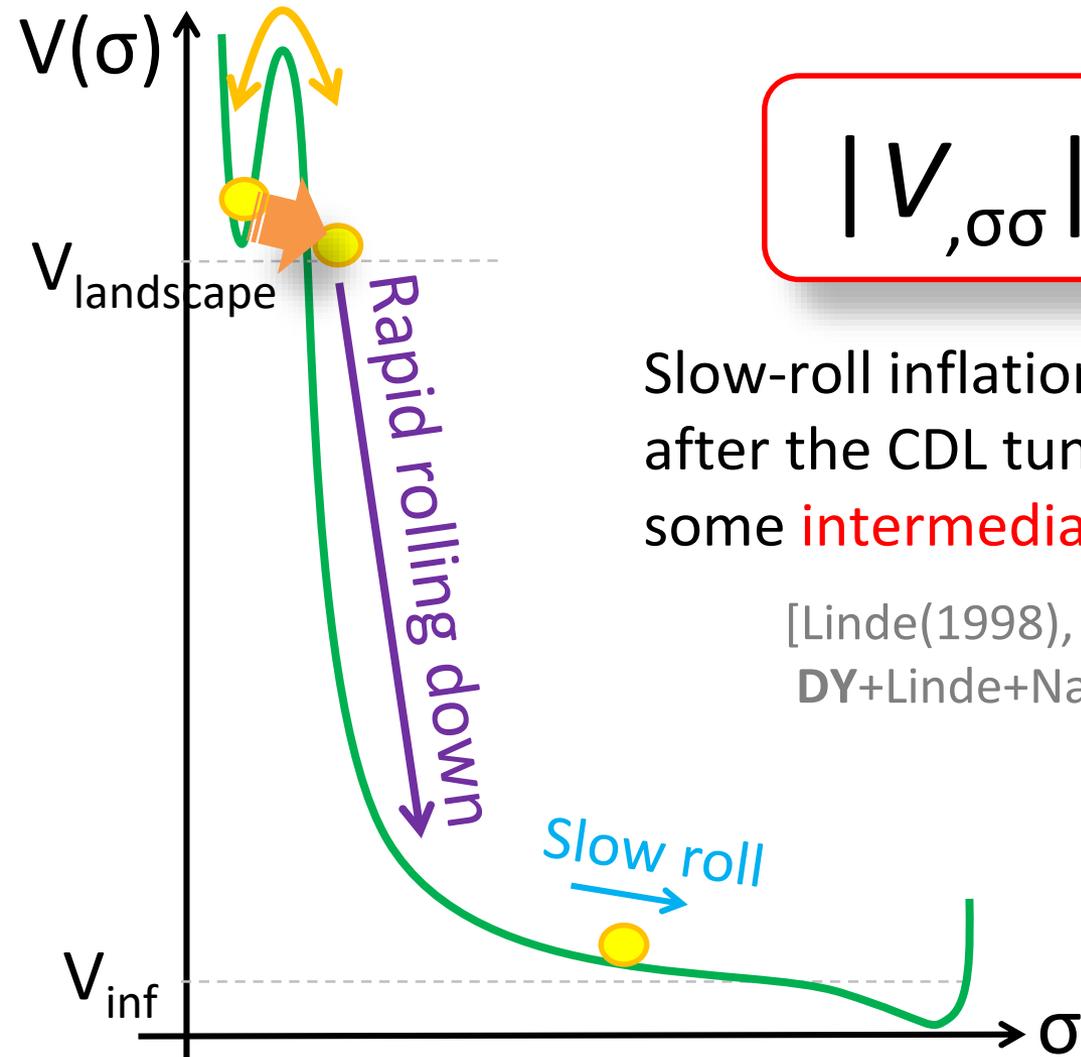
$$V_{\text{landscape}} = O(M_{\text{pl}}^4) \gg V_{\text{inf}}$$

The universe will most likely tunnel to a point where the energy scale is still very high and have a **large hierarchy between energy scales**.

Ex.) Stringy inflation  $V_{\text{inf}} = O(10^{-30} M_{\text{pl}}^4)$

[Kallosh+Linde(2014)]

# Requirements for a single-field stringy model



$$|V_{,\sigma\sigma}| \gg V/M_{\text{Pl}}^2$$

Slow-roll inflation does not start immediately after the CDL tunneling, and there must be some **intermediate stage of rapid rolling down**.

[Linde(1998), Linde+Sasaki+Tanaka(1999),  
DY+Linde+Naruko+Sasaki+Tanaka, 1105.2674]

# Tensor-type perturbations

may not be suppressed at all !

$$\langle |h|^2 \rangle \approx \frac{2}{M_{\text{pl}}^2} \left( \frac{H}{2\pi} \right)^2$$

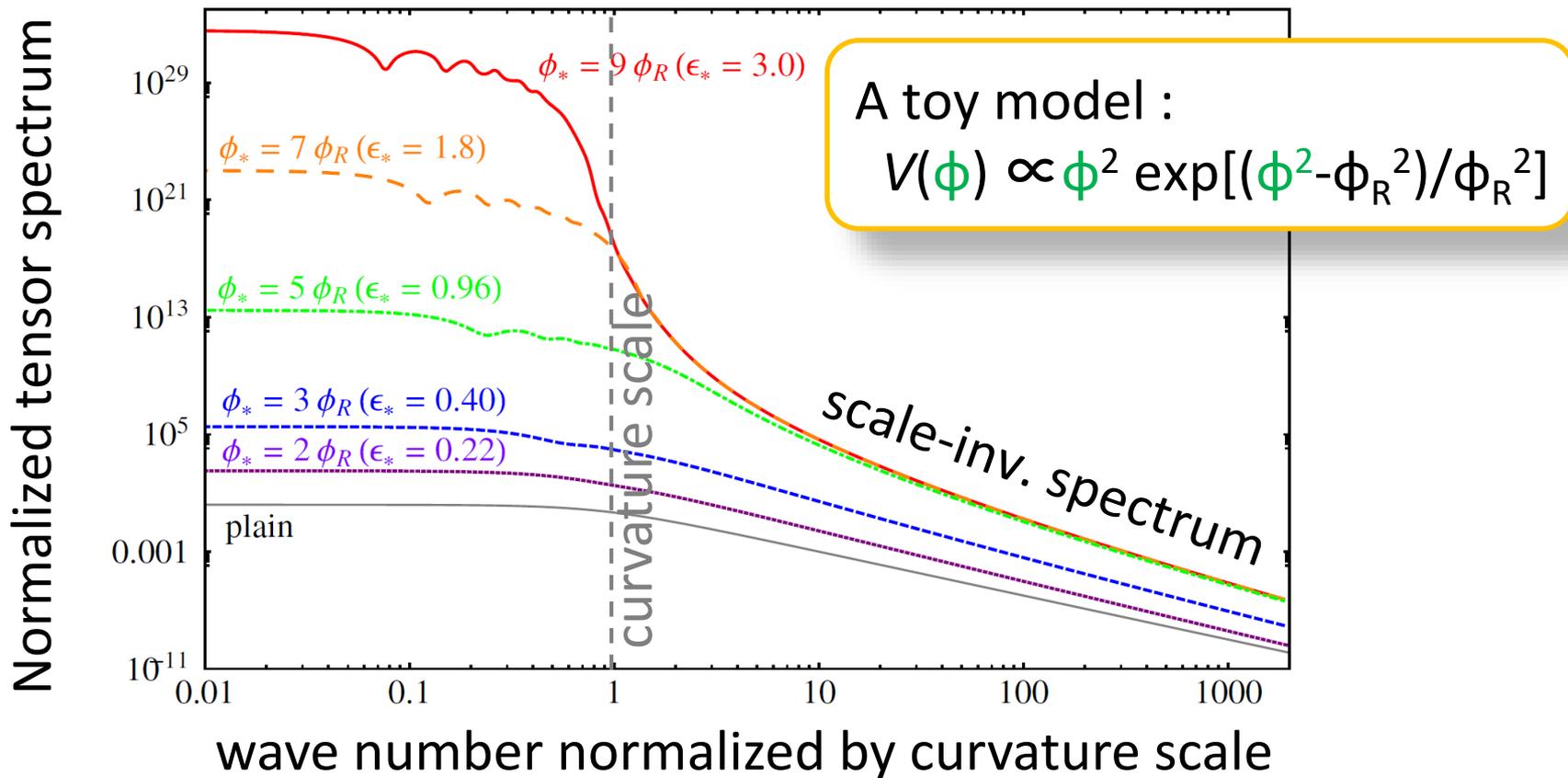
Memory of false vacuum may remain in the perturbations on the curvature scale!

- ✓ Note : Scalar perturbations may be suppressed by the velocity during RAPID ROLL phase...

$$\langle |\mathcal{R}^2| \rangle \approx \left( \frac{H^2}{2\pi \dot{\phi}} \right)^2$$

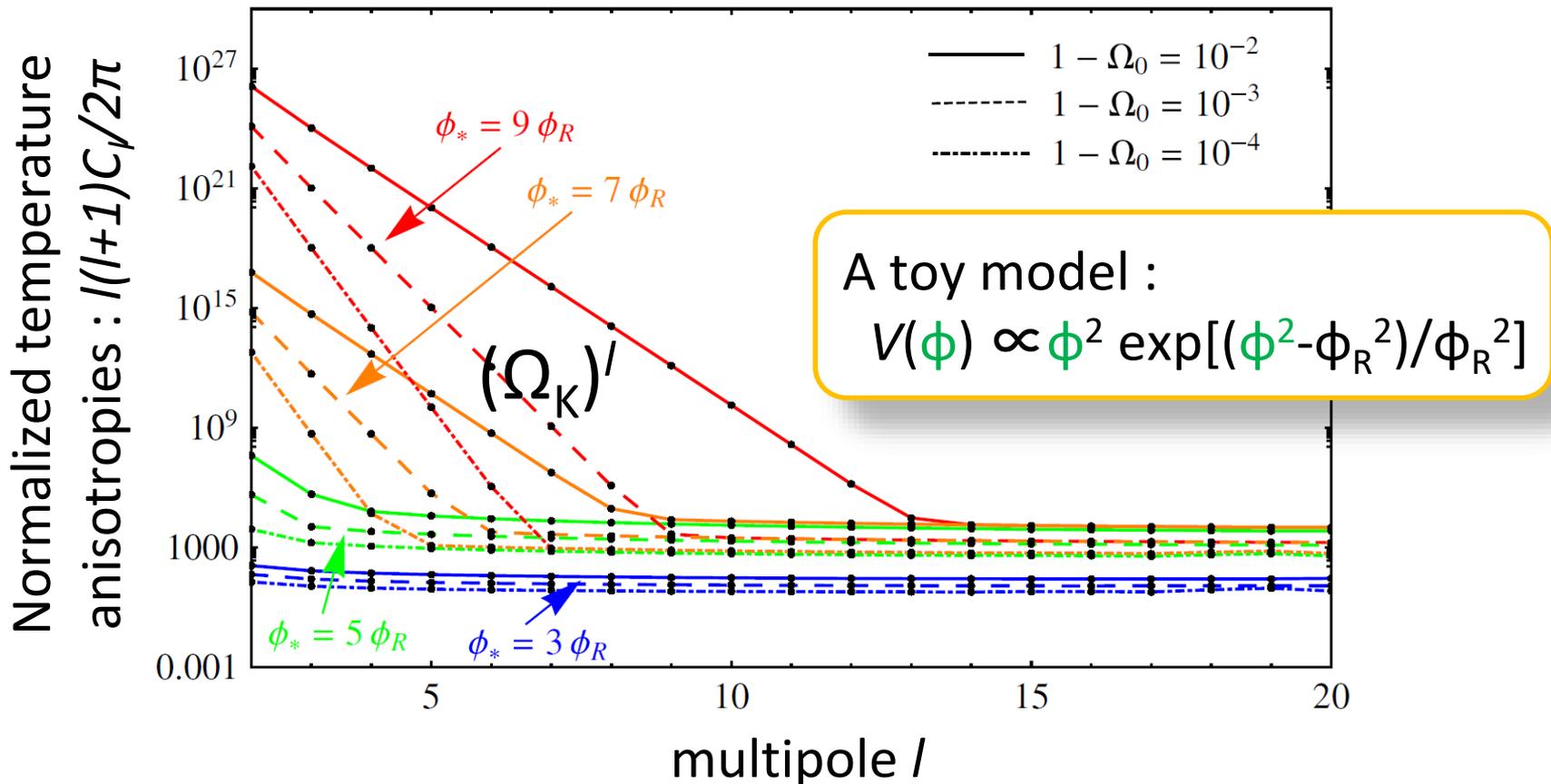
# Primordial tensor spectrum

Keeps the memory of the high energy density in large angular scales, and can be **strongly red-tilted**.



# Primordial tensor spectrum

Keeps the memory of the high energy density in large angular scales, and can be **strongly red-tilted**.



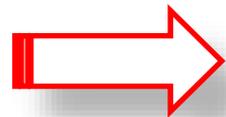
Part 4

# **ASYMMETRY FROM OPEN INFLATION**

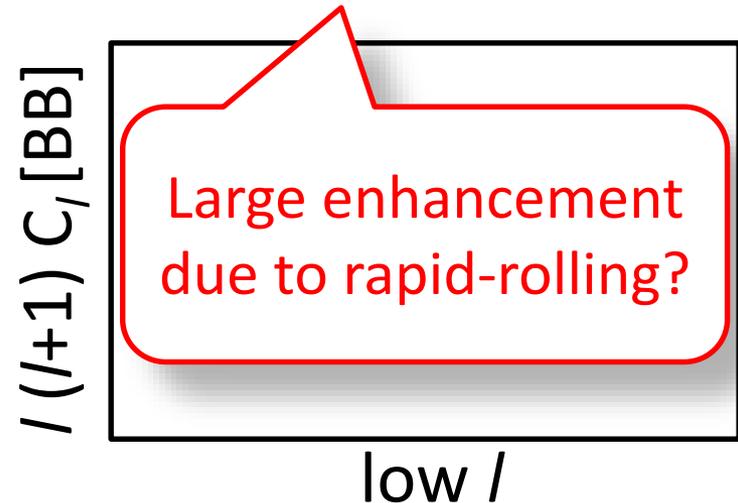
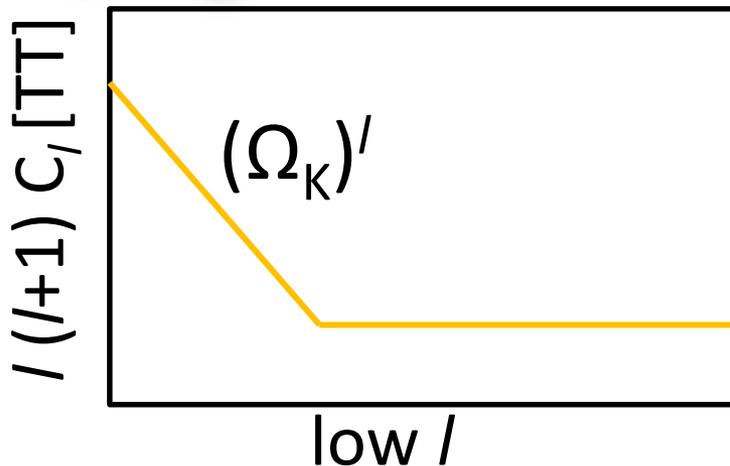
# Polarizations from open inflation

- Spatially openness :  $\Omega_K=10^{-2}-10^{-4}$
- A complete set of mode functions
  - ✓ (Continuous) normal modes
  - ✓ (Discrete) supercurvature modes

implement in  
CAMB code



CMB temperature + **polarizations**

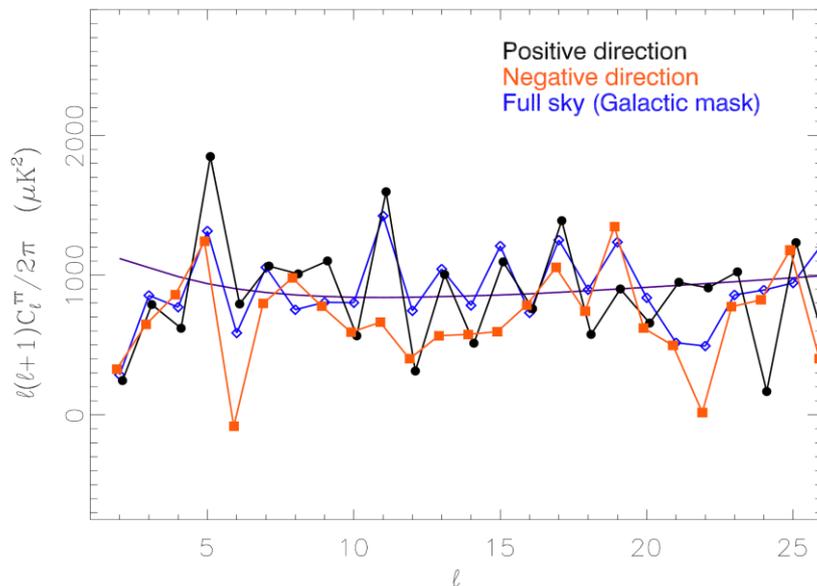


# CMB anomalies

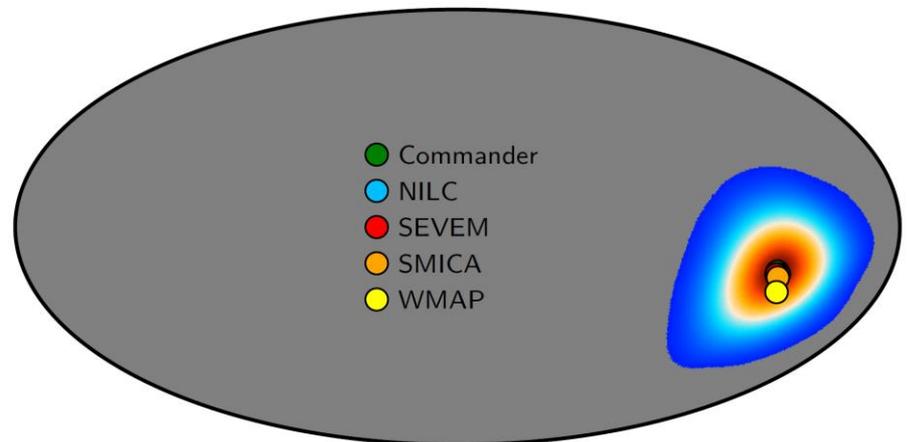
$$\Delta T(\mathbf{n}) = (1 + A \mathbf{p} \cdot \mathbf{n}) \Delta T_{\text{iso}}(\mathbf{n})$$

$$A = 0.07 \pm 0.02$$

## Power asymmetry



## Dipole modulation



# Modulation due to supercurvature

[see also Kanno+Sasaki+Tanaka(2013), Bernes+Domenech+Sasaki+Takahashi(2016)]

We will treat the **supercurvature mode** as a ***nonstochastic quantity*** and we can only observe one realization in our Hubble patch.

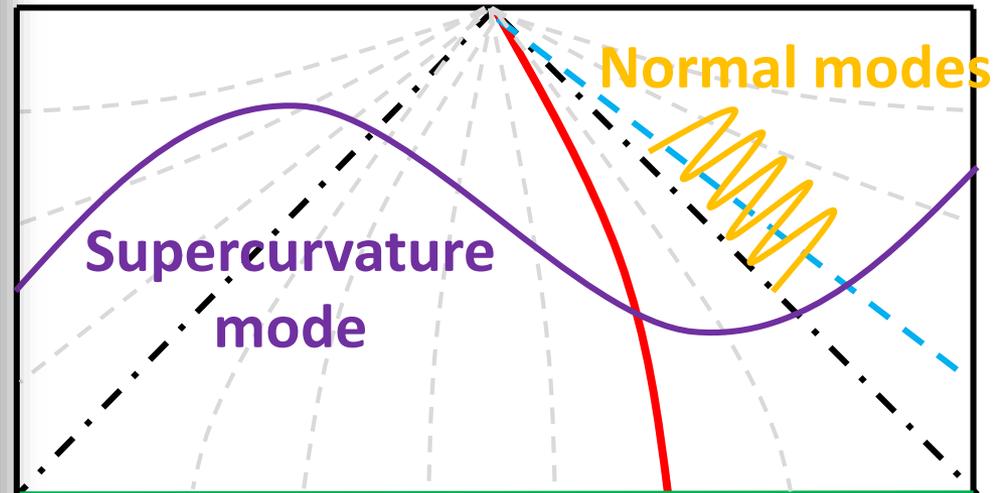
➡ The modulation of the continuous spectrum

Eternal inflating region

Hubble patch

Normal modes

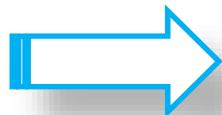
Supercurvature mode

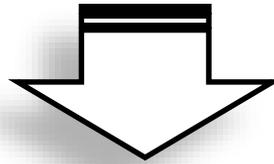


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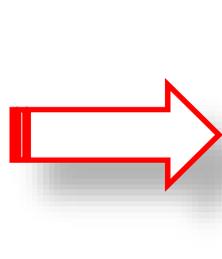
We will treat the **supercurvature mode** as a *nonstochastic quantity* and we can only observe one realization in our Hubble patch.

 **The modulation of the continuous spectrum**



$$\mathcal{R}_c(\mathbf{x}) = \mathcal{N} \Big|_{\sigma_{\text{bg}}} - \mathcal{N} \Big|_{\sigma_{\text{bg}} + \Delta\sigma(\mathbf{x})} - \mathcal{N}_\sigma \Big|_{\sigma_{\text{bg}} + \Delta\sigma(\mathbf{x})} \delta\sigma(\mathbf{x}) - \frac{1}{2} \mathcal{N}_{\sigma\sigma} \Big|_{\sigma_{\text{bg}} + \Delta\sigma(\mathbf{x})} \delta\sigma^2(\mathbf{x}) + \dots$$

**Spatial modulation due to SC mode!**

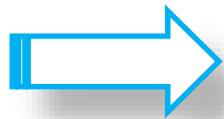
-  **Power asymmetry :  $A = \Delta P_R / 2P_R$**
- Quadrupole :  $a_{20}$**

[DY+Zarei+Hirouzjahi+Ohta+Naruko+Yamaguchi, work in progress]

# Modulation due to supercurvature

[see also Kanno+Sasaki+Tanaka(2013), Bernes+Domenech+Sasaki+Takahashi(2016)]

We will treat the **supercurvature mode** as a ***nonstochastic quantity*** and we can only observe one realization in our Hubble patch.



**The modulation of the continuous spectrum**

***Hemispherical power asymmetry!***



# Modulation due to supercurvature

[see also Kanno+Sasaki+Tanaka(2013), Bernes+Domenech+Sasaki+Takahashi(2016)]

We will treat the **supercurvature mode** as a *nonstochastic quantity* and we can only observe one realization in our Hubble patch.

## Question...

- ✓ Does the supercurvature mode with the large amplitude really exist in the realistic open inflationary scenario?

Preliminary

# Future prospects

## ➤ Multifield tunneling and supercurvature mode

- ✓ Background solution [Sugimura+DY+Sasaki (2011)]
- ✓ Quantization including the effect of the metric perturbations should be taken into account.
- ✓ would rescue the simplest two field model [Tanaka+Sasaki(1996)]

$$L=(M_{pl}^2/2)R-G_{ab}(\boldsymbol{\varphi})d\varphi^a \cdot d\varphi^b/2-V(\boldsymbol{\varphi})$$

 supercurvature modes?

# Future prospects

## ➤ Higher-order correlations

- ✓ Bispectrum from scalar **normal** modes in exact dS  
[Sugimura+Komatsu (2013)]

$$B_{\zeta}^{\text{subcurv}} \simeq B_{\zeta}^{\text{usual}} + B_{\zeta}^{\text{NBD}} \quad (\text{squeezed limit, subcurvature limit})$$

$$\left\{ \begin{array}{l} B_{\zeta}^{\text{usual}} \rightarrow (1-n_s) P_{\zeta}(k_{\text{long}}) P_{\zeta}(k_{\text{short}}) \\ B_{\zeta}^{\text{NBD}} \rightarrow \underbrace{(k_{\text{short}}/k_{\text{long}})}_{\text{enhancement}} \underbrace{\exp(-\pi k_{\text{short}})}_{\text{suppression}} P_{\zeta}(k_{\text{long}}) P_{\zeta}(k_{\text{short}}) \end{array} \right.$$

- ✓ Extension to the dynamical background is needed.
- ✓ Contributions from **supercurvature modes**@ $k \sim k_{\text{superlong}}$ ?

# Summary

*We are already testing the model of inflation  
in the context of cosmic/string landscape !*

