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Collisions of superconducting strings with Y-junctions

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世界へ、そして未来へ
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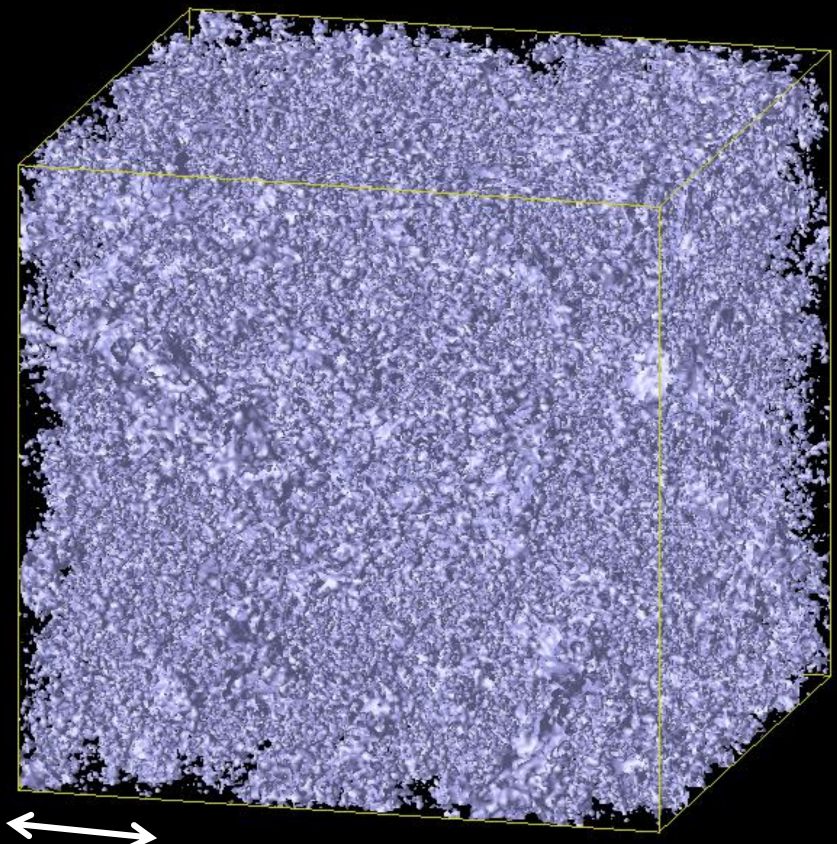
Cosmic string network



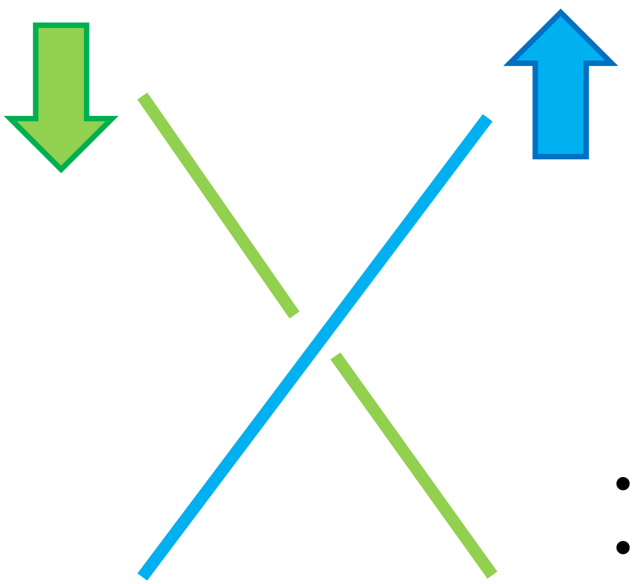
Hubble length when simulation starts.

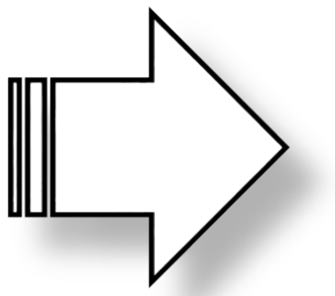
- Evolves according to *scaling*.
- Eventually loses its energy through loop formation due to *partner exchange* : *reconnection*.
- Although the reconnection almost always happens, strings with *Y-typed junctions* occurs in many models!

Hiramatsu+Sendouda+Takahashi+**DY**+Yoo, 1307.0308

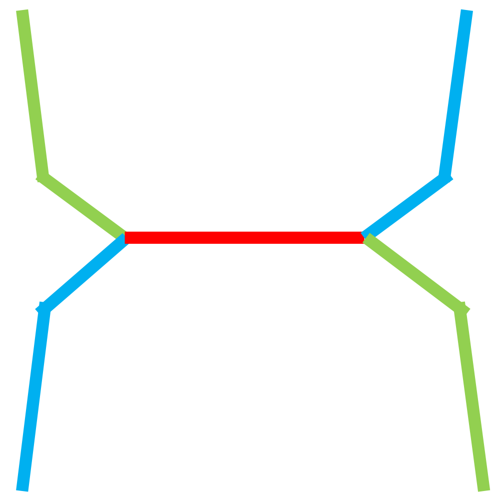
Y-junction formation?

When two strings collide, bound states and Y-junctions would be produced.





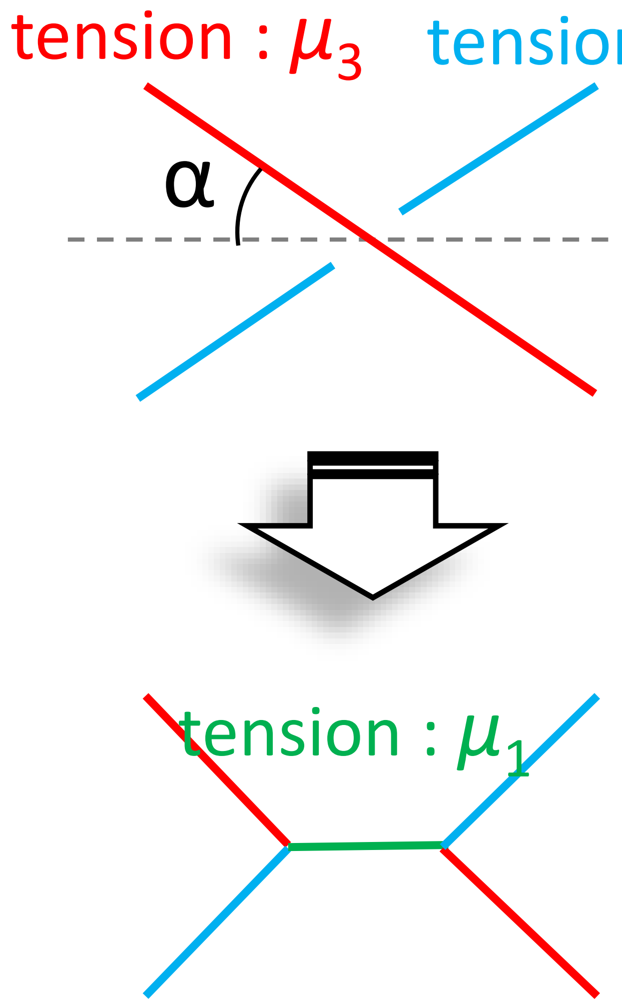
- Nearly parallel
- Low relative velocity

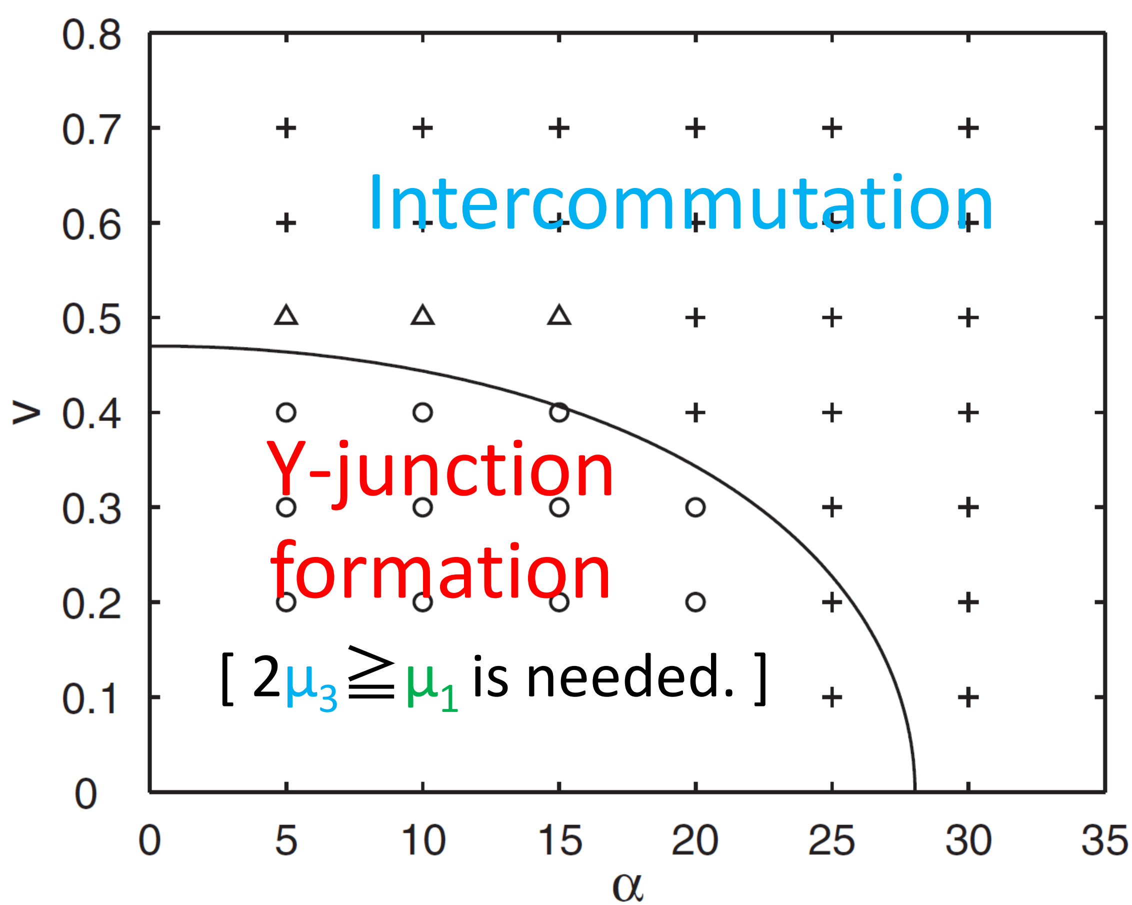


[Copeland+Kibble+Steer(2006)(2007), Salmi+Achucarro+Copeland+de Putter+Steer(2008)]

Y-junction formation : *condition*

$$\alpha < \arccos\left(\frac{\mu_1}{2\mu_3\sqrt{1-v^2}}\right)$$





[Copeland+Kibble+Steer(2006),(2007)]

Strategy [Nambu-Goto case]

① Consider effective action

$$S = -\mu \int \mathrm{d}\tau \mathrm{d}\sigma \sqrt{-\gamma_{ab}}$$

Choose gauge conditions

② Extend ① to include multiple strings with junctions

$$S = -\sum_j \mu_j \int \mathrm{d}\tau \mathrm{d}\sigma \, \Theta(s_j(\tau) - \sigma) \sqrt{-\gamma_j}$$

$$+ \sum_j \int \mathrm{d}\tau \mathfrak{f}_j \cdot \left[x_i(s_j(\tau), \tau) - X(\tau) \right]$$

Junction position at $\sigma = s_j(\tau)$

4-dim embedding of a junction

③ Solve junction conditions

Questions we are interested in here are:

- **What happens to the currents when two *current-carrying* strings collide?**
- **Can junctions actually form?**

We would like to look for the condition for the junction formation in the analytic way (if possible).

Differences between Nambu-Goto and current-carrying strings

Conformal and temporal gauges

- ***Worldsheet gauge choices*** generally made to study NG ***DO NOT*** apply to general elastic strings

  ⇒ We need to develop a *fully covariant* formalism.

- Equations of motion are generally ***NOT integrable***

  [exception : chiral strings]

  ⇒ We need to solve *junction conditions* for string position and current *simultaneously*.

Step 1 : effective action

- Lagrangian depends on its internal degree of freedom φ :

$$S = \int d^2\sigma \sqrt{-\det(\gamma_{ab})}\, \mathcal{L}(w)$$

[Carter(1989a,b)]

- ✓ *L* is constant → Nambu-Goto strings
- ✓ *L* is a function of *state parameter w* → superconducting strings

[Witten(1985)]

$$w \equiv \kappa_0 \gamma^{ab} \varphi_{,a} \varphi_{,b}$$

[For superconducting strings, φ characterizes a phase of scalar field living on a string.]

Energy-momentum tensor

- Nambu-Goto strings

Conformal and temporal gauges

$$T^{\mu\nu} = U\, u^{\mu}u^{\nu} - T\, v^{\mu}v^{\nu}$$

$$\Rightarrow \quad U = T \qquad u^{\mu} \propto X^{\mu}{}_{,\tau}$$

[For simplicity, w>0]

Energy-momentum tensor

- Current-carrying strings

Conformal ~~and temporal~~ gauges

$$T^{\mu\nu} = U\, u^\mu u^\nu - T\, v^\mu v^\nu$$

⇨ $U \neq T$ $\qquad u^\mu \propto \varphi^{,a} x^\mu{}_{,a}$

✓ ***NOTICE :*** One can use the *residual* freedom of Lorentz rotation on the worldsheet!

⇨ $\varphi(\tau,\sigma) \rightarrow \varphi(\sigma)$

Preferred rest frame

[Nambu-Goto case : Copeland+Kibble+Steer(2006)(2007)]

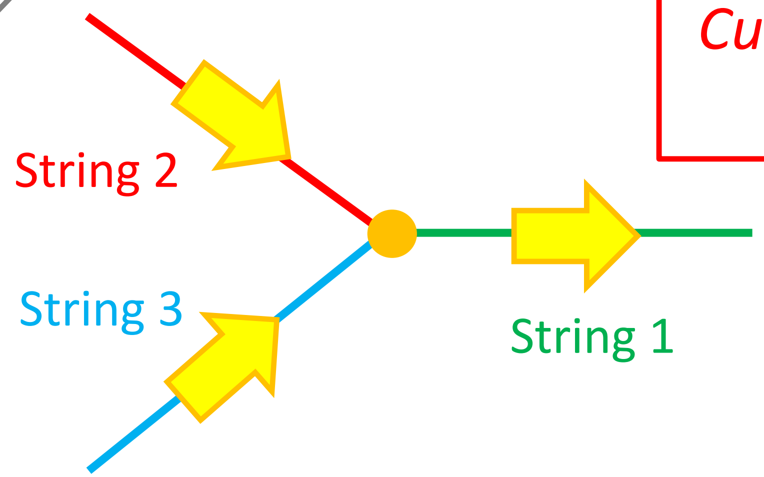
Step 2 : Effective action with a junction

Junction position at $\sigma = s_j(\tau)$

$$S = \sum_i \int d\tau\, d\sigma\, \theta\left(s_i(\tau) - \sigma\right) \sqrt{-\gamma_i}\, \mathcal{L}(w_i)$$
$$+ \sum_i \int d\tau \left\{ \mathfrak{f}_i^\mu \cdot \left[x_{i,\mu}(s_i(\tau),\tau) - X_\mu(\tau)\right] + \mathfrak{g}_i \cdot \left[\varphi_i(s_i(\tau),\tau) - \Phi(\tau)\right] \right\}$$

4-dim embedding of a junction

Current conservation at a junction



[Carter+Steer+Lilley+**DY**+Hiramatsu in progress]

Step 3 : Covariant form of junction conditions

NEW

	Current-carrying strings	Nambu-Goto strings
Energy-momentum	$\sum_j \Gamma_j^2 \left(U_j \dot{s}_j^2 - T_j\right) \lambda_j^\mu = 0$ $\sum_j \Gamma_j^2 \dot{s}_j \left(U_j - T_j\right) = 0$	$\sum_j T_j \lambda_j^\mu = 0$ Outward-directed unit normal vector (λ_j^μ)
Current	$\sum_j \Gamma_j \nu_j \dot{s}_j = 0$ [w>0] $\sum_j \Gamma_j \nu_j = 0$ [w<0]	

[Carter+Steer+Lilley+**DY**+Hiramatsu in progress]

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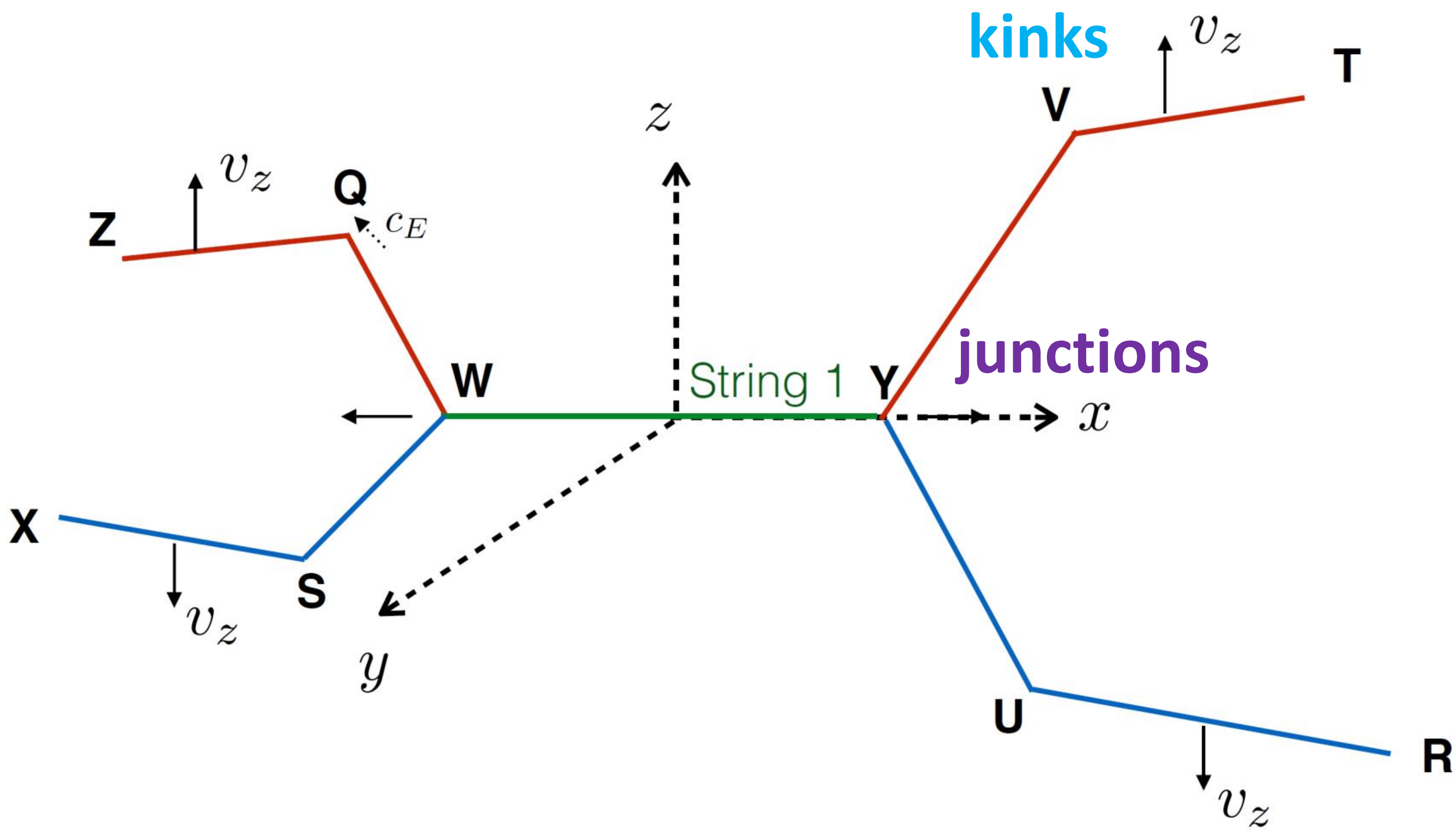
Current	$\sum_j \Gamma_j \nu_j \dot{s}_j = 0$ [*w*>0] $\sum_j \Gamma_j \nu_j = 0$ [*w*<0]	

2 more constraint equations!

Step 4 : Apply to string collision

Consider a string collision between 2 incoming and identical strings at angle $\pm\alpha$ with velocity $\pm v_z$:



Step 4.1 : Nambu-Goto case

- Junction conditions
- Unknowns

$$\sum_j T_j \lambda_j^\mu = 0 \quad \Leftrightarrow \quad \begin{cases} \dot{s}_3 = \dot{s}_2 = -\dfrac{\mu_3}{2\mu_1}\dot{s}_1 \\ \dot{s}_1 = \dfrac{2\mu_3\sqrt{1-v_z^2}\cos\alpha - \mu_1}{2\mu_3 - \mu_1\sqrt{1-v_z^2}\cos\alpha} \end{cases}$$

All unknowns can be determined by the junction conditions!

⇨ The solution makes sense only if s1>0 :
the connecting string cannot get shorter.

$$\alpha < \arccos\left(\frac{\mu_1}{2\mu_3\sqrt{1-v^2}}\right)$$

Step 4.2 : Current-carrying case

➢ Junction conditions

$$\begin{cases} \sum_j \Gamma_j^2 \left(U_j \dot{s}_j^2 - T_j\right) \lambda_j^\mu = 0 \\ \sum_j \Gamma_j^2 \dot{s}_j \left(U_j - T_j\right) = 0 \\ \sum_j \Gamma_j \nu_j \dot{s}_j \ = \ 0 \end{cases}$$

2 more constraint eqs!

⟺

➢ Unknowns

$$\begin{cases} \dot{s}_3 = \dot{s}_2 = \dfrac{c_{\mathrm{E}} - v_+}{1 - c_{\mathrm{E}} v_+} \\ \dot{s}_1 = \cdots \\ w_1 = \cdots \end{cases}$$

Step 4.2 : Cur

In the presence of the current, $s_3=s_2$ is determined ***NOT*** by the junction conditions but by the configuration.

- Junction conditions

$$\left\{\begin{array}{l} \sum_j \Gamma_j^2 \left(U_j \dot{s}_j^2 - T_j\right) \lambda_j^\mu = 0 \\ \sum_j \Gamma_j^2 \dot{s}_j \left(U_j - T_j\right) = 0 \\ \sum_j \Gamma_j \nu_j \dot{s}_j = 0 \end{array}\right. \Leftrightarrow \left\{\begin{array}{l} \dot{s}_3 = \dot{s}_2 = \dfrac{c_{\mathrm{E}} - v_+}{1 - c_{\mathrm{E}} v_+} \\ \dot{s}_1 = \cdots \\ w_1 = \cdots \end{array}\right.$$

2 more constraint eqs!

There are only **2** unknown variables!

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$$\left\{\begin{aligned} &\sum_j \Gamma_j^2 \left(U_j \dot{s}_j^2 - T_j\right) \lambda_j^\mu = 0 \\ &\sum_j \Gamma_j^2 \dot{s}_j \left(U_j - T_j\right) = 0 \\ &\sum_j \Gamma_j \nu_j \dot{s}_j \ = \ 0 \end{aligned}\right. \Leftrightarrow \left\{\begin{aligned} &\dot{s}_3 = \dot{s}_2 = \frac{c_\mathrm{E} - v_+}{1 - c_\mathrm{E} v_+} \\ &\dot{s}_1 = \cdots \\ &w_1 = \cdots \end{aligned}\right.$$

2 more constraint eqs!

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(# of eqs to solve) **>** (# of unknowns)

The system is ***OVERDETERMINED***!

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➢ Junction conditions

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$$\dot{s}_3 = \dot{s}_2 = \frac{c_\mathrm{E} - v_+}{1 - c_\mathrm{E} v_+}$$

The joining string can *NOT* be described by the elastic model (that is, a barotropic EoS).

$$S = \int d^2\sigma \sqrt{-\det(\gamma_{ab})}\, \mathcal{L}(w)$$

bles!

(# of eqs to solve) (# of unknowns)

The system is ***OVERDETERMINED***!

Summary

- We have extended the analysis of the subsequent formation of Y-junction to the elastic models characterizing current-carrying strings.
- There are big differences between NG and elastic strings:
  - ✓ Gauge issues
  - ✓ Presence of internal DoF

In the case of the string collision, the joining string can NOT be described by the elastic model.

Future prospects

- The treatment of such a collision may generally require the use of a ***non-conservative model***.
- It is of great interest to compare the results from the ***numerical investigation*** of a collision of current-carrying strings.

→ Takashi Hiramatsu's talk

Thank you!