2019/11/20 Theoretical aspects of non-Gaussianity from modern perspectives@YITP

Constraining Primordial non-Gaussianity with future galaxy surveys

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Current CMB constraint on PNG

◆Planck 2018

$f_{NL}^{local} = -0.9 \pm 5.1$
$f_{NL}^{equil} = -26 \pm 47$
$f_{NL}^{ortho} = -38 \pm 24$
$g_{NL}^{local} = (-5.8 \pm 6.5) \times 10^4$

CMB experiments are already close to **CV-limited** ones.

Experiments	f_{NL} (bispectrum)	f_{NL} (skewness)
COBE	600	800
MAP	20	80
Planck	5	70
Ideal	3	60

[Komatsu +Spergel(2000)]

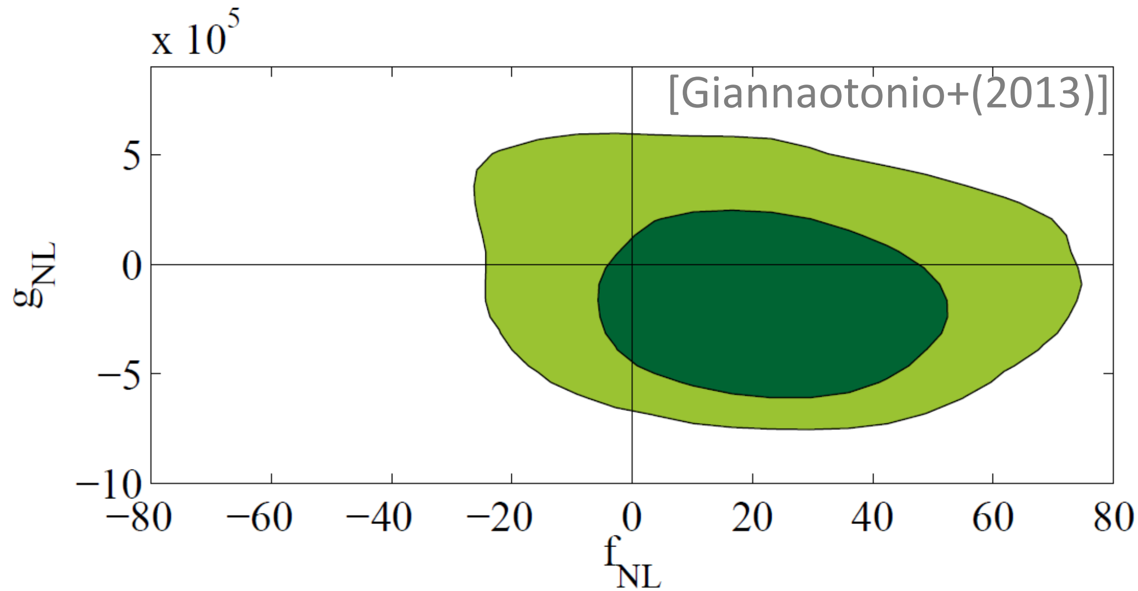
Current LSS constraint on PNG

◆Galaxy **power** spectrum

f_{NL}^{local} = -113 +-154 (1σ) [Ho+(2013)]

f_{NL}^{local} = 5 +- 21 (1σ) [Giannaotonio+(2013)]

f_{NL}^{local} = -15+-36 (2σ) [Castorina+(2019)]

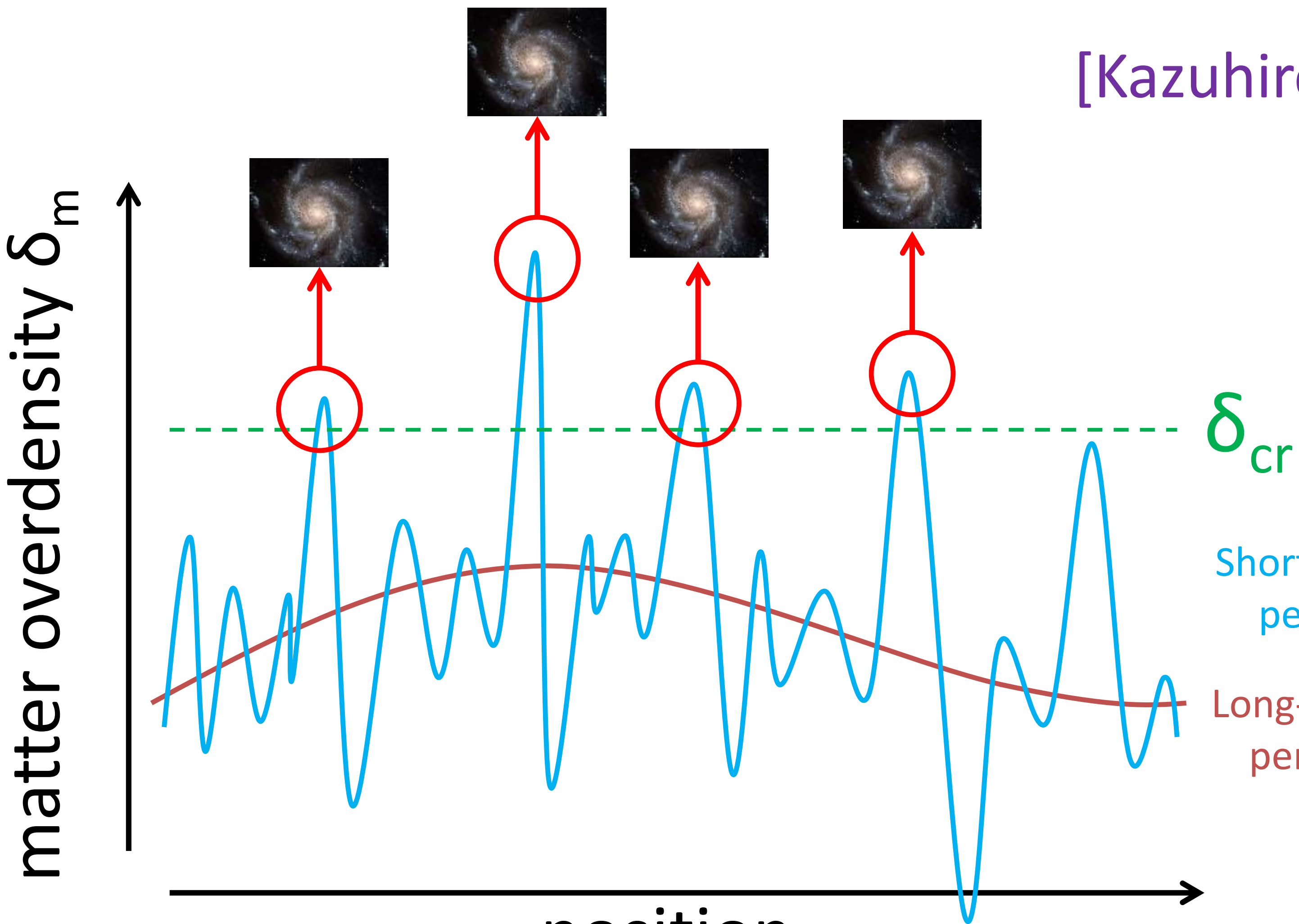


How do we constrain local PNG with galaxy surveys?

Galaxy bias

Luminous sources should be treated as **biased** tracer of LSS.

[Kazuhiro's talk]



Galaxy bias

Galaxy number density contrast δ_{gal} does not exactly coincide with matter overdensity field δ_m, but …

$$\delta_{gal} = F[\delta_m, \ldots]$$

$$= b_1 \, \delta_m + b_2 \, \delta_m{}^2/2 + \ldots$$

Linear bias   Nonlinear bias

(which is naturally induced by nonlinear gravitational evolution)

✓ In the Gaussian case, the bias on large scales is scale-invariant.

f_{NL}^{local} induces scale-dependence(1)

[Dalal+Dore+Huterer+Shirokov(2008)]

◆"Rough" estimation [Kazuhiro's talk]

$$\Phi_{NG} = \phi_G + f_{NL}(\phi_G)^2$$

Acting the Laplacian on Φ_{NG}...

$$\underbrace{\nabla^2\Phi_{NG}}_{\propto\ \delta_{NG}} = \underbrace{\nabla^2\phi_G}_{\propto\ \delta} + 2f_{NL}\underbrace{\phi_G}_{\propto\ \delta/k^2}\underbrace{\nabla^2\phi_G}_{\propto\ \delta} + \cdots$$

$$\Rightarrow \quad \delta_{NG} \simeq (1 + 2f_{NL}\phi_G)\delta$$

f_{NL}^{local} induces scale-dependence(2)

[Smith+LoVerde(2011), Smith+Ferraro+LoVerde(2011)]

◆To be precise, let us decompose the potential into

$$\Phi = \phi_{short} + \phi_{long}$$

[Yuichiro's talk]

$$\Rightarrow \Delta_{\Phi} = (1 + 2f_{NL}\phi_{long})\, \Delta_{\Phi short} + \ldots$$

◆The halo number density on large scales is given by

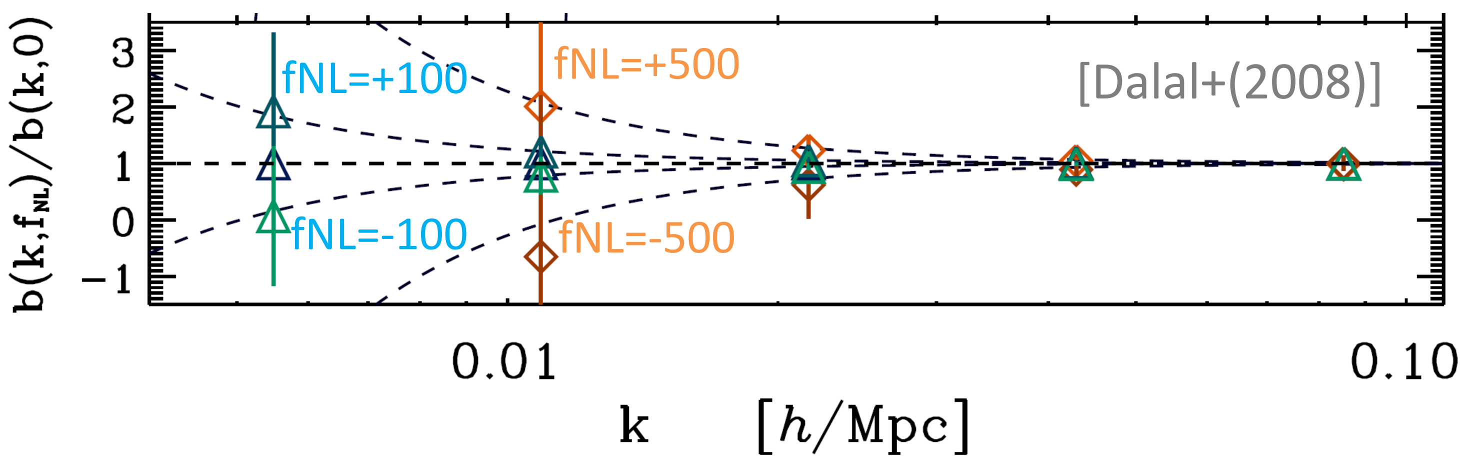
$$n_{halo} = n_{bg} + (\mathrm{d}n/\mathrm{d}\delta_{long})\, \delta_{long}$$
$$+ 2f_{NL}(\mathrm{d}n/\mathrm{d}\Delta_{\Phi})\, \phi_{long} + \ldots$$

f_{NL}^{local} induces scale-dependence(3)

- Local-type PNG induces $\Delta b \propto 1/k^2$ dependence such that the effect dominates at very large scales:

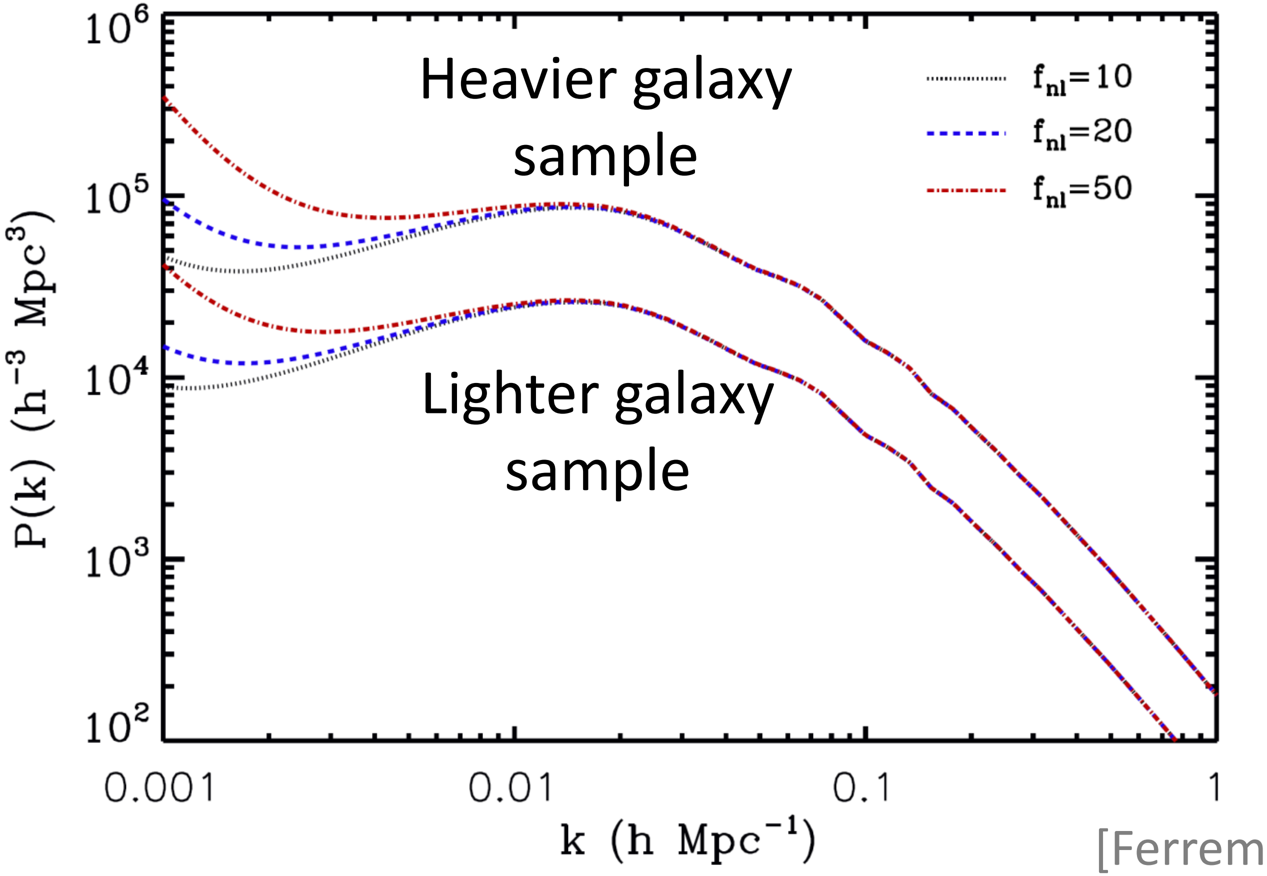
[Dalal+(2008), Desjacques+(2009)]

$$\Delta b = \frac{3 H_0^2 \Omega_{m,0} \delta_c (b_G - 1)}{k^2 T(k) D_+(z)} f_{NL}$$



f_{NL}^{local} induces scale-dependence(3)

$$P_{gg} = (b_1 + b_{NG}^{(f)} f_{NL}^{local}/k^2)^2 P_{mm}$$



[Ferremacho+(2014)]

f_{NL}, g_{NL} and τ_{NL}

$$f_{NL}^{local} \rightarrow \Delta b = b_{NG}^{(f)} f_{NL}^{local} / k^2$$

$$g_{NL}^{local} \rightarrow \Delta b = b_{NG}^{(g)} g_{NL}^{local} / k^2$$

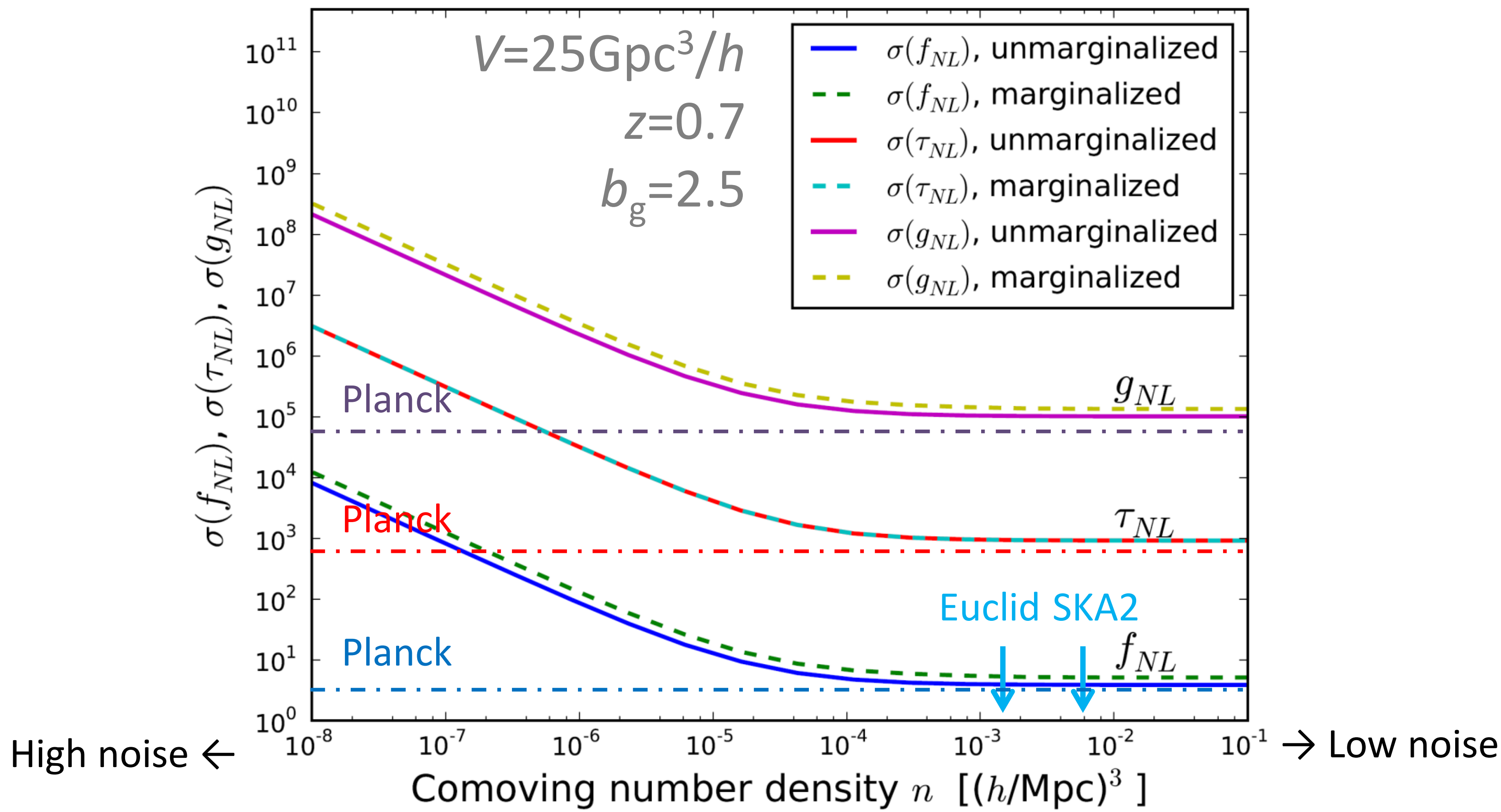
- $f_{NL} + \tau_{NL}$ case

$$P_{gg} = \left[(b_1 + b_{NG}^{(f)} f_{NL}/k^2)^2 + \left\{ (5/6)^2 \tau_{NL} - f_{NL}^{\,2} \right\} (b_{NG}^{(f)})^2 / k^4 \right] P_{mm}$$

$\geqq 0$ (Suyama-Yamaguchi inequality)

Constraining local PNG with galaxy survey

[Ferraro+Smith(2014)]

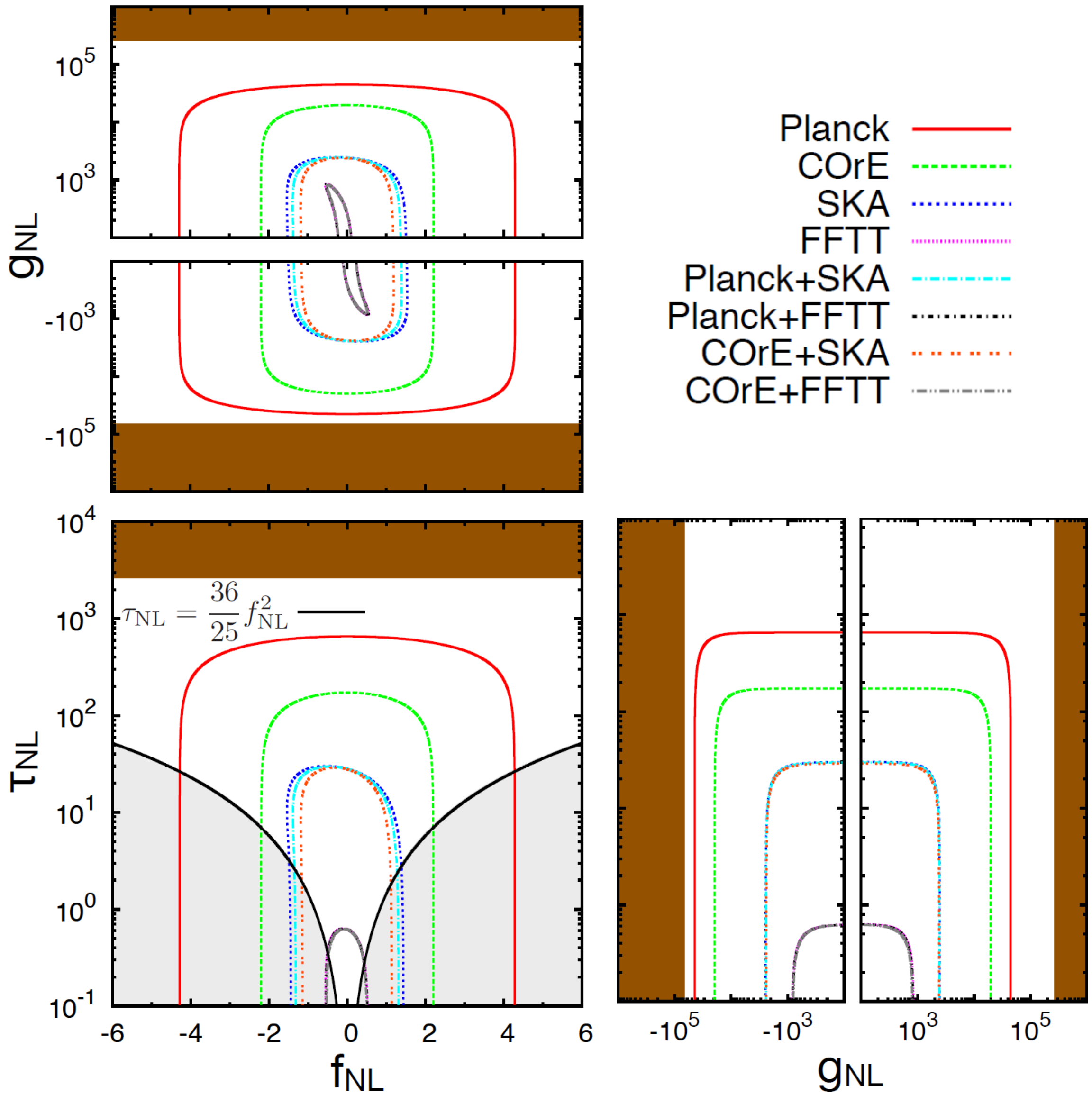


[Sekiguchi+Takahashi+Tashiro+Yokoyama(2018)]

Constraining local PNG with minihalos

$\sigma(g_{NL})/10^3$=
- 18 [COrE(CMB)]
- 2.3 [SKA]
- 0.79 [FFTT]

$\sigma(\tau_{NL})$=
- 160 [COrE(CMB)]
- 28 [SKA]
- 0.59 [FFTT]



f_{NL}^{equil}, f_{NL}^{fold} and f_{NL}^{ortho}

[e.g., Matsubara(2012)]

$f_{NL}^{local} \rightarrow \Delta b \propto f_{NL}^{local} / k^2$

Relatively weak k-dependence

$f_{NL}^{equil} \rightarrow \Delta b \propto f_{NL}^{equil} / k^0$

$f_{NL}^{fold} \rightarrow \Delta b \propto f_{NL}^{fold} / k^1$

$f_{NL}^{ortho} \rightarrow \Delta b \propto f_{NL}^{ortho} / k^1$

It is difficult to constrain non-local PNG with scale-dep bias…

[Yoo(2010), Challinor+Lewis(2011), Bruni+(2011),...]

Note: Relativistic corrections

- The **observed** galaxy overdensity on superhorizon scales depends on the gravitational potentials:

$$\Delta(\mathbf{n},z) \equiv \frac{N(\mathbf{n},z)-\langle N\rangle(z)}{\langle N\rangle(z)} = \frac{\rho(\mathbf{n},z)-\bar{\rho}(z)}{\bar{\rho}(z)} + \frac{\delta V(\mathbf{n},z)}{V(z)}$$

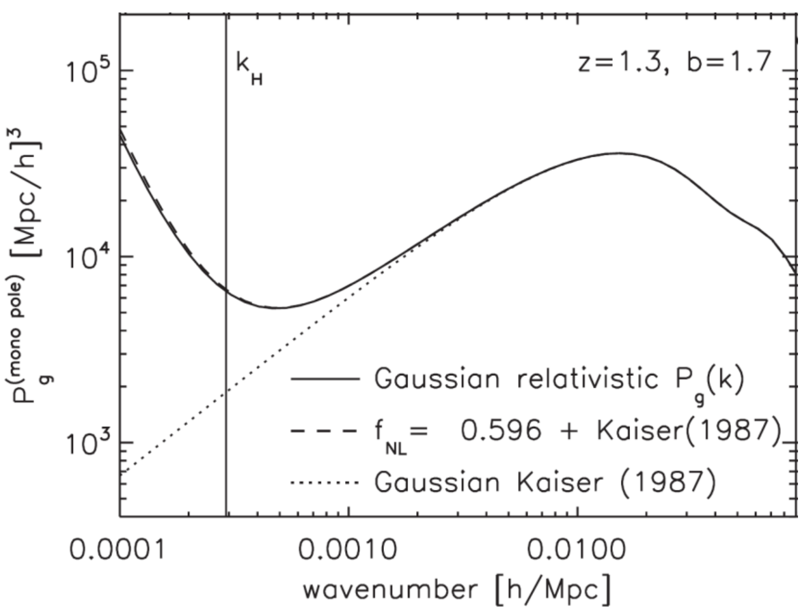
$$= \underbrace{\delta - \frac{1}{\mathcal{H}}\mathbf{n}\cdot\frac{\partial \mathbf{v}}{\partial \chi}}_{\text{standard}} + \underbrace{(5s-2)\kappa}_{\text{lensing}} + \underbrace{\left(\frac{1}{\mathcal{H}}\Psi' + \Phi + \cdots\right)}_{\text{GR effect}}$$

$\propto \delta/k^2$ (Poisson eq.)

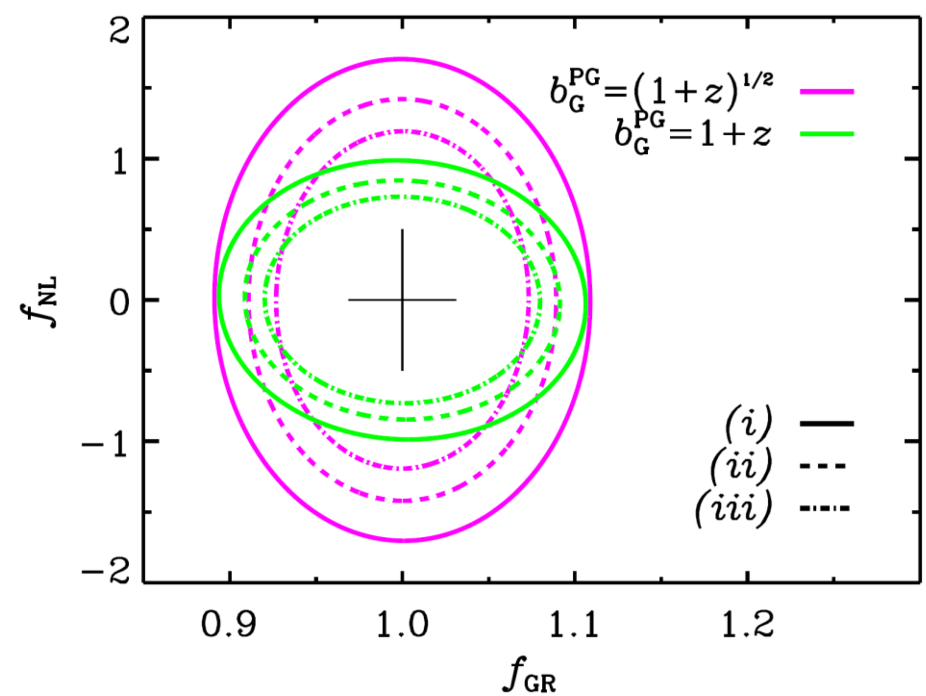
In order not to bias the estimation of PNG, it will be crucial to correctly model the relativistic corrections !

Note: Relativistic corrections

[Jeong+Schmidt+Hirata(2011)]



[Fonseca+(2015)]



There is a degeneracy between f_{NL}^{local} and GR effects.

Multi-tracer cosmology

Multi-tracer cosmology

Cosmic variance: fundamental limit to large-scale obs.

- Only a finite number of Fourier modes in our Hubble volume
- Biased populations probe the **same** DM field → **deterministic**
- Tracer-dependent quantities are no CV-limited [Seljak(2008)]

Tracer1 $\delta_1 = (b_1 + f\mu^2)\,\delta_{DM}$

Tracer2 $\delta_2 = (b_2 + f\mu^2)\,\delta_{DM}$

Tracers are **stochastic**

$$\delta_1/\delta_2 = (b_1 + f\mu^2)/(b_2 + f\mu^2)$$

Ratio is **deterministic**

Noise on MT quantities scales like shot noise

→ Need high source density & large bias ratio

Multi-tracer cosmology

- Error on cosmological parameter θ

$$\sigma(\theta) = (F_{\theta\theta})^{-1/2} = (dP/d\theta \cdot \mathrm{Cov}[P,P]^{-1} \cdot dP/d\theta)^{-1/2}$$

- Single tracer case

$$\sigma(\ln P) = (P \cdot (P+N^{-1})^{-2} \cdot P)^{-1/2} \rightarrow 1+O(1/PN)$$

Even when $N\rightarrow\infty$,
finite error remains.
:Cosmic Variance

[Seljak (2009), Hamaus+Seljak+Desjacques(2011)]

Multi-tracer cosmology

All galaxy samples
$N_{tot,}$
$\delta_{gal} = b_{tot}\ \delta$

⇒

Light galaxies
N_1
$\delta_{gal,1} = b^{(1)}\ \delta$

Heavy galaxies
N_2
$\delta_{gal,2} = b^{(2)}\ \delta$

$$\sigma(\theta) = (F_{\theta\theta})^{-1/2} = (\ d\boldsymbol{P}/d\theta \cdot \mathrm{Cov}[\boldsymbol{P},\boldsymbol{P}]^{-1} \cdot d\boldsymbol{P}/d\theta)^{-1/2}$$

[Seljak (2009), Hamaus+Seljak+Desjacques(2011)]

Multi-tracer cosmology

All galaxy samples
$N_{tot,}$
$\delta_{gal} = b_{tot}\ \delta$

Light galaxies
N_1
$\delta_{gal,1} = b^{(1)}\ \delta$

Heavy galaxies
N_2
$\delta_{gal,2} = b^{(2)}\ \delta$

$$\boldsymbol{P} = \{ P^{(11)}, P^{(12)}, P^{(22)} \} = \{ \alpha^2 P_2, r\alpha P_2, P_2 \}$$

$$\sigma(\theta) = (F_{\theta\theta})^{-1/2} = (d\boldsymbol{P}/d\theta \cdot \mathrm{Cov}[\boldsymbol{P},\boldsymbol{P}]^{-1} \cdot d\boldsymbol{P}/d\theta)^{-1/2}$$

$$\mathrm{Cov} = \begin{pmatrix} (\alpha^2P_2+N_1^{-1})^2 & (\alpha^2P_2+N_1^{-1})r\alpha P_2 & r^2\alpha^2P_2^2 \\ * & ((\alpha^2P_2+N_1^{-1})(P_2+N_2^{-1})+r^2\alpha^2P_2^2)/2 & (P_2+N_2^{-1})r\alpha P_2 \\ * & * & (P_2+N_2^{-1})^2 \end{pmatrix}$$

Multi-tracer cosmology

All galaxy samples
$N_{tot,}$
$\delta_{gal} = b_{tot}\,\delta$

Light galaxies
N_1
$\delta_{gal,1} = b^{(1)}\,\delta$

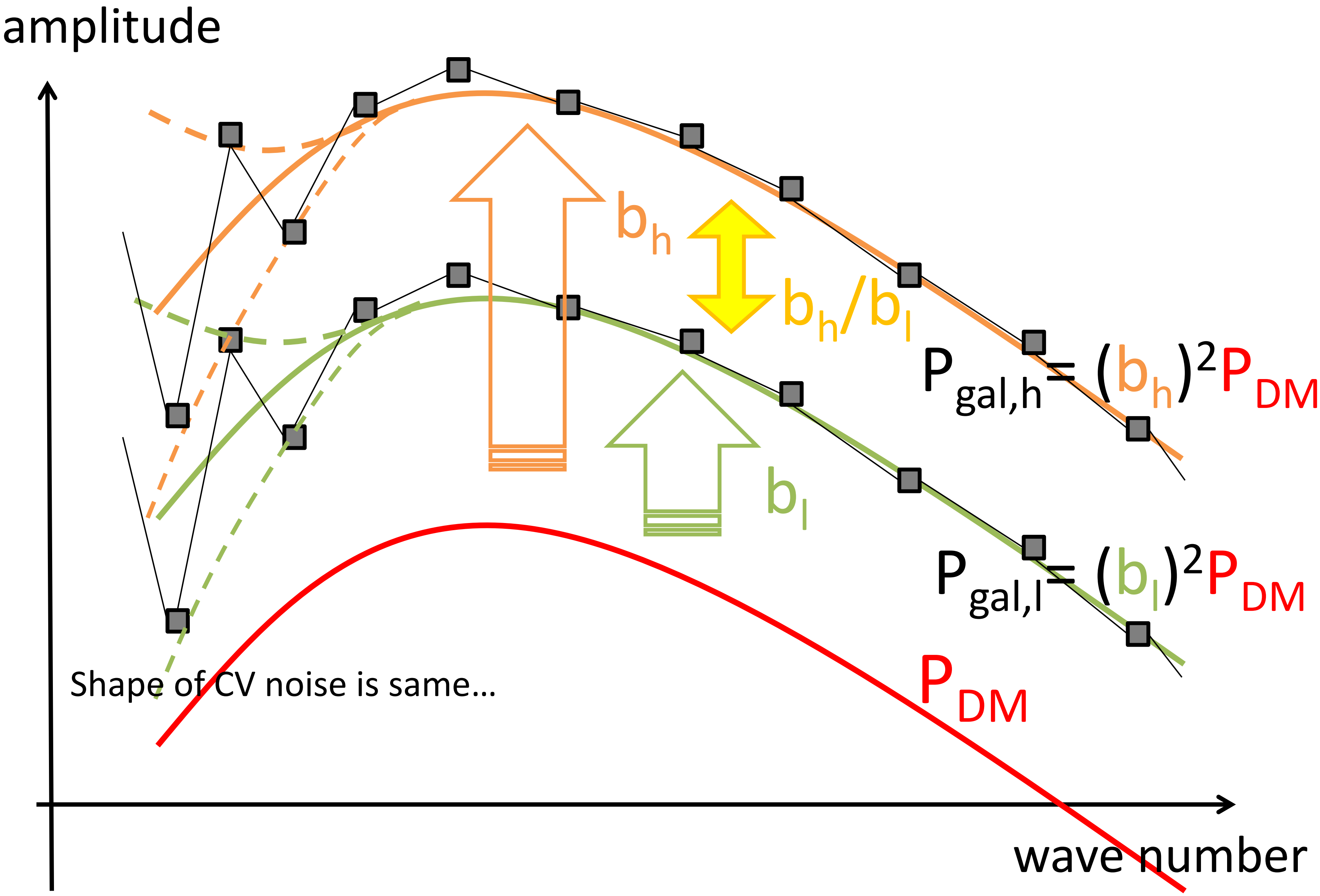
Heavy galaxies
N_2
$\delta_{gal,2} = b^{(2)}\,\delta$

$$\boldsymbol{P} = \{ P^{(11)}, P^{(12)}, P^{(22)} \} = \{ \alpha^2 P_2, r\alpha P_2, P_2 \}$$

$$\sigma(\alpha) = (F_{\alpha\alpha})^{-1/2} = (d\boldsymbol{P}/d\alpha \cdot \mathrm{Cov}[\boldsymbol{P},\boldsymbol{P}]^{-1} \cdot d\boldsymbol{P}/d\alpha)^{-1/2}$$

$$\sigma(\alpha = b_1^{(1)}/b_1^{(2)}) \rightarrow ((P_2 N_1)^{-1} + \alpha^2 (P_2 N_2)^{-1})^{1/2}$$

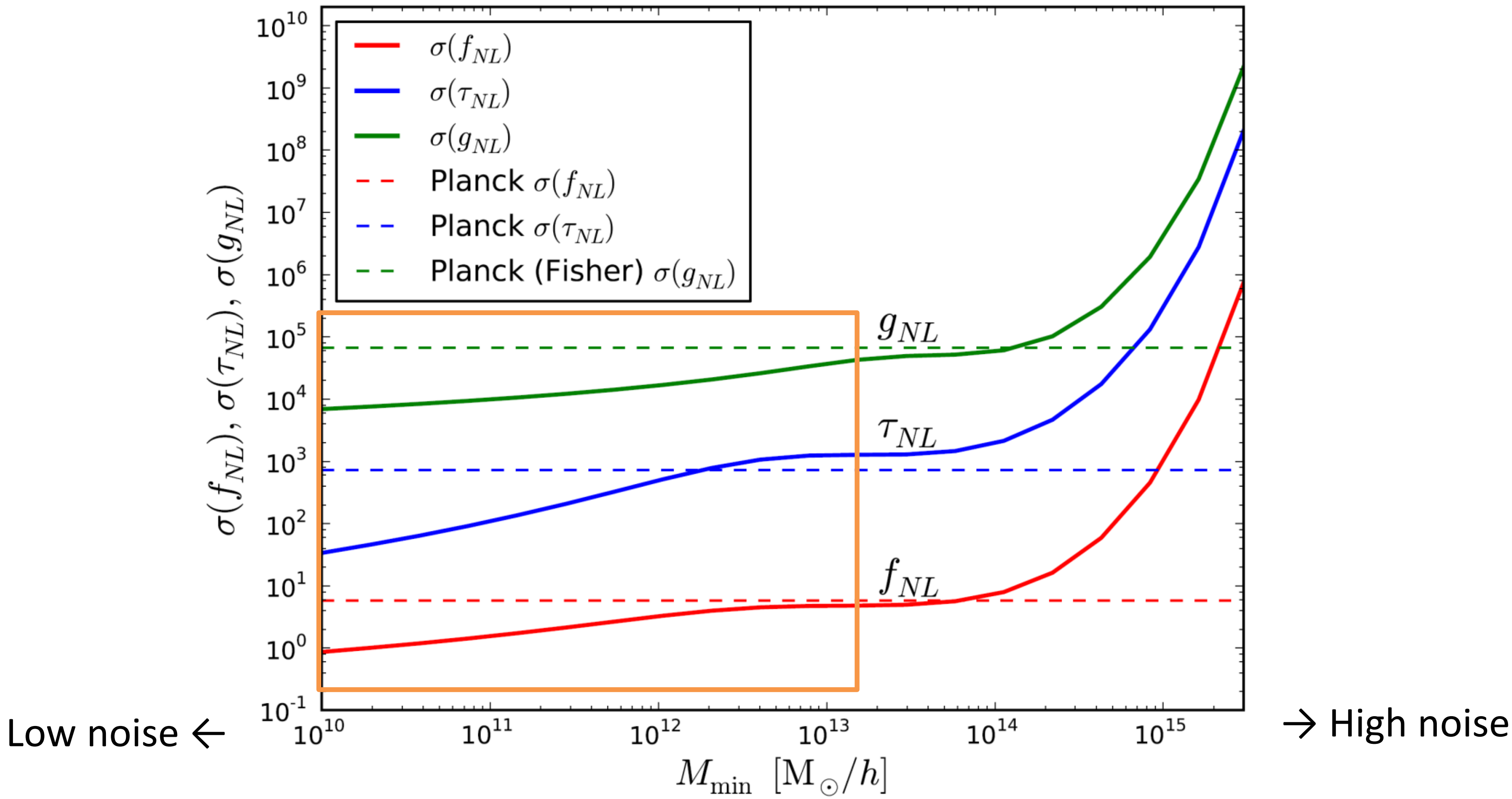
Error on **ratio** is determined only by shot noise!


amplitude
b_h
b_h/b_l
P_gal,h = (b_h)^2 P_DM
b_l
P_gal,l = (b_l)^2 P_DM
P_DM
Shape of CV noise is same…
wave number

Constraining power of MT

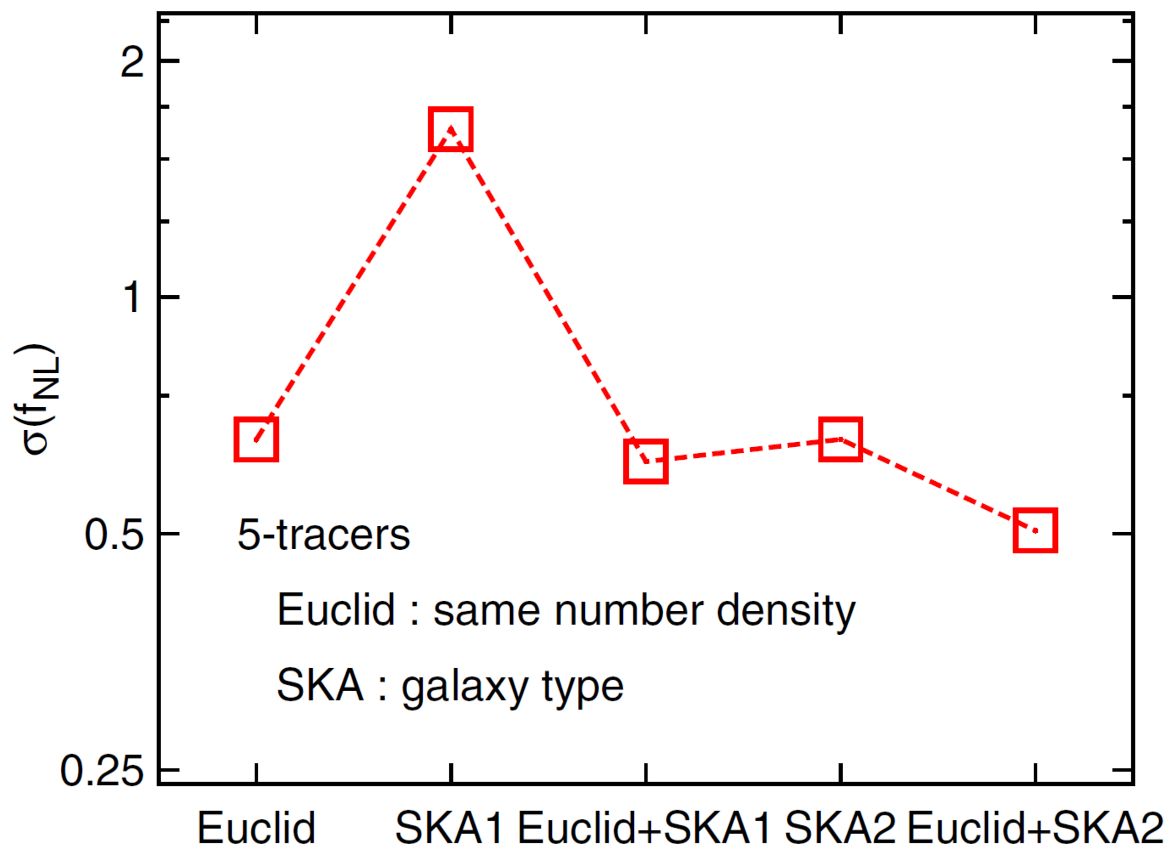
[Ferraro+Smith(2014)]

MT is **effective** for low shot-noise region.



Constraining f_{NL}^{local} with SKA+Euclid

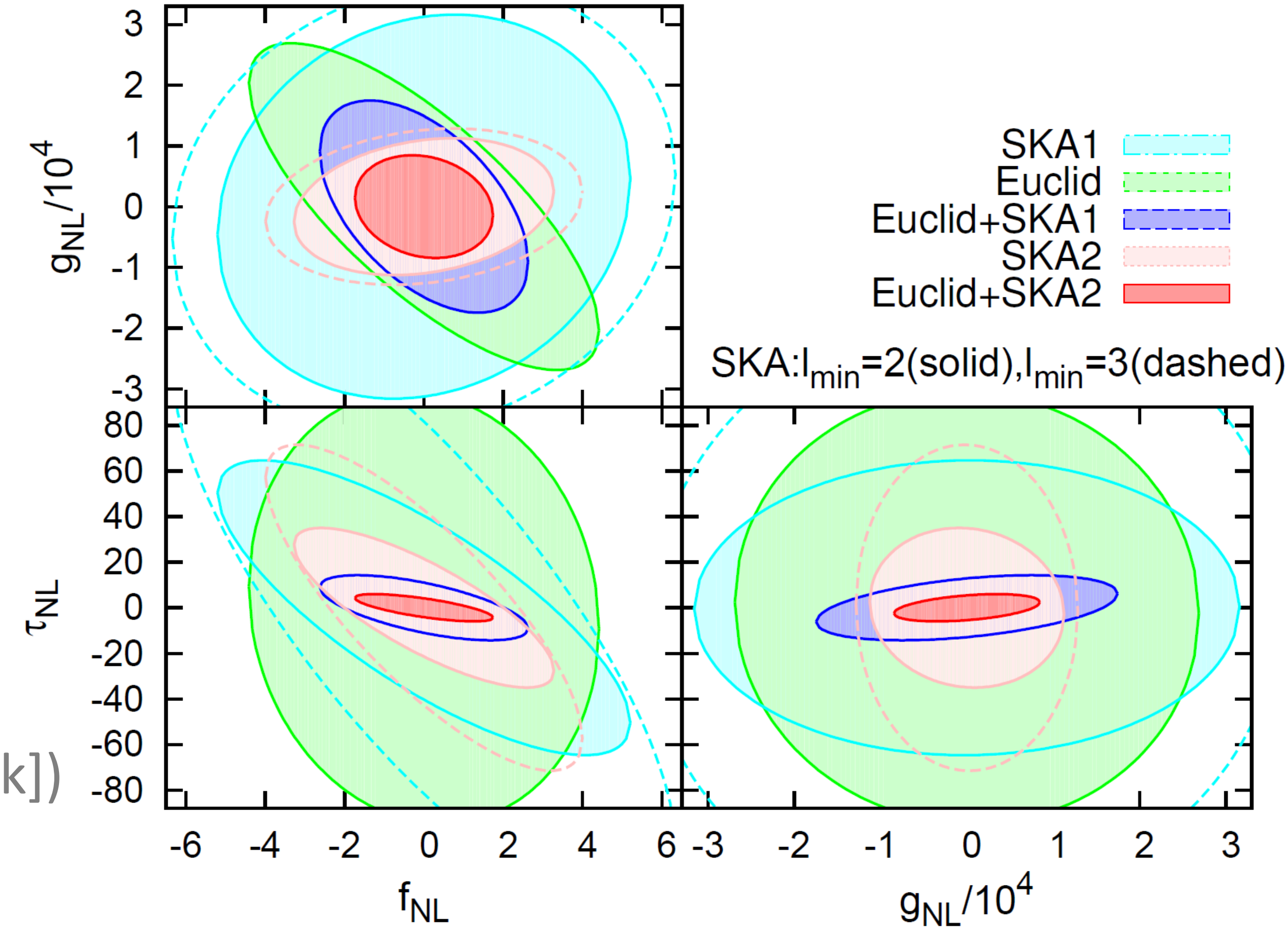
[**DY**+Takahashi+Oguri(2014)]



Constraining local PNG with SKA+Euclid

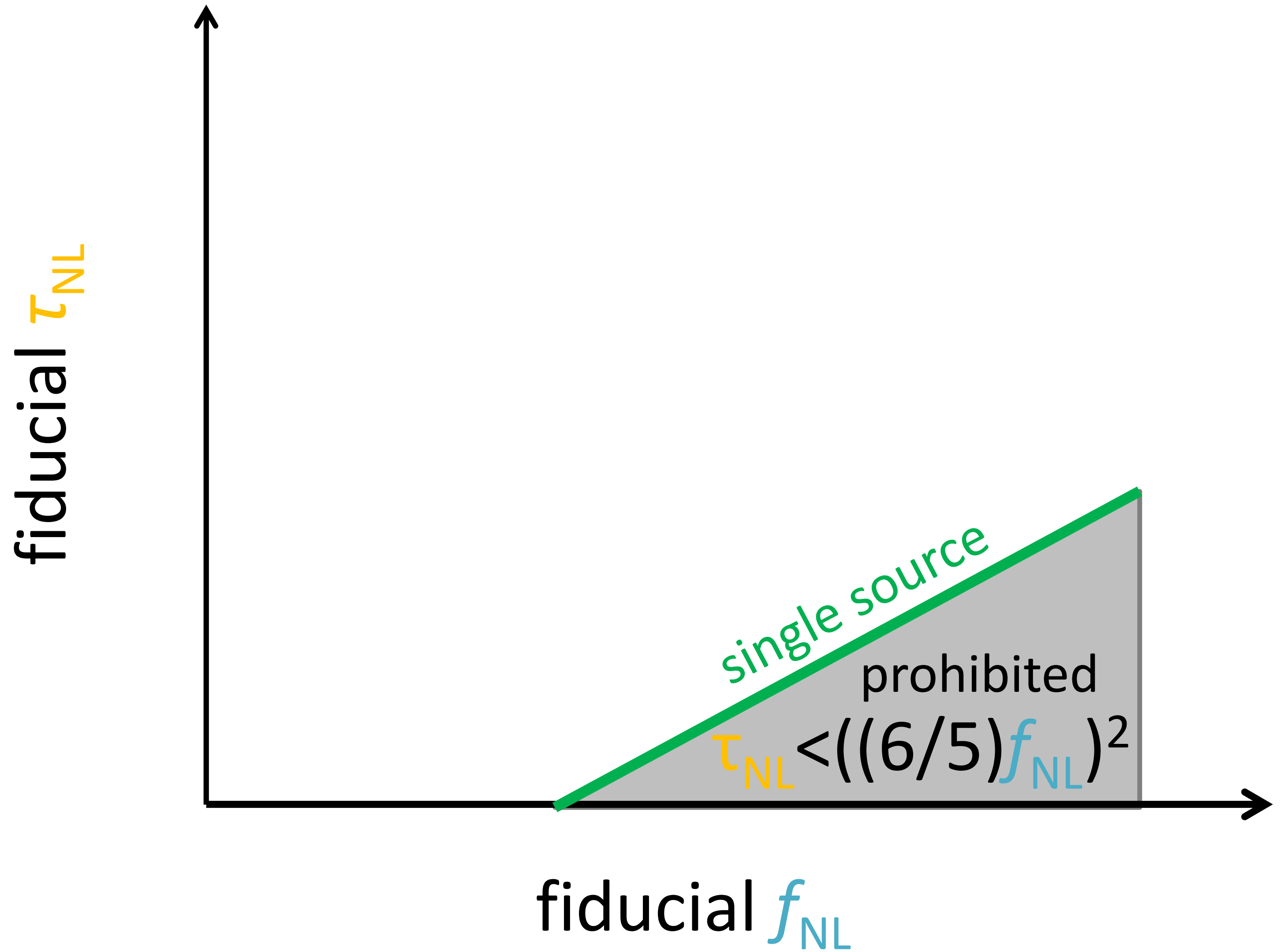
[DY+Takahashi(2015)]

$\sigma(g_{NL})/10^3$=
21 [SKA1]
7.4 [SKA2]
18 [Euclid]
(65 [Planck])



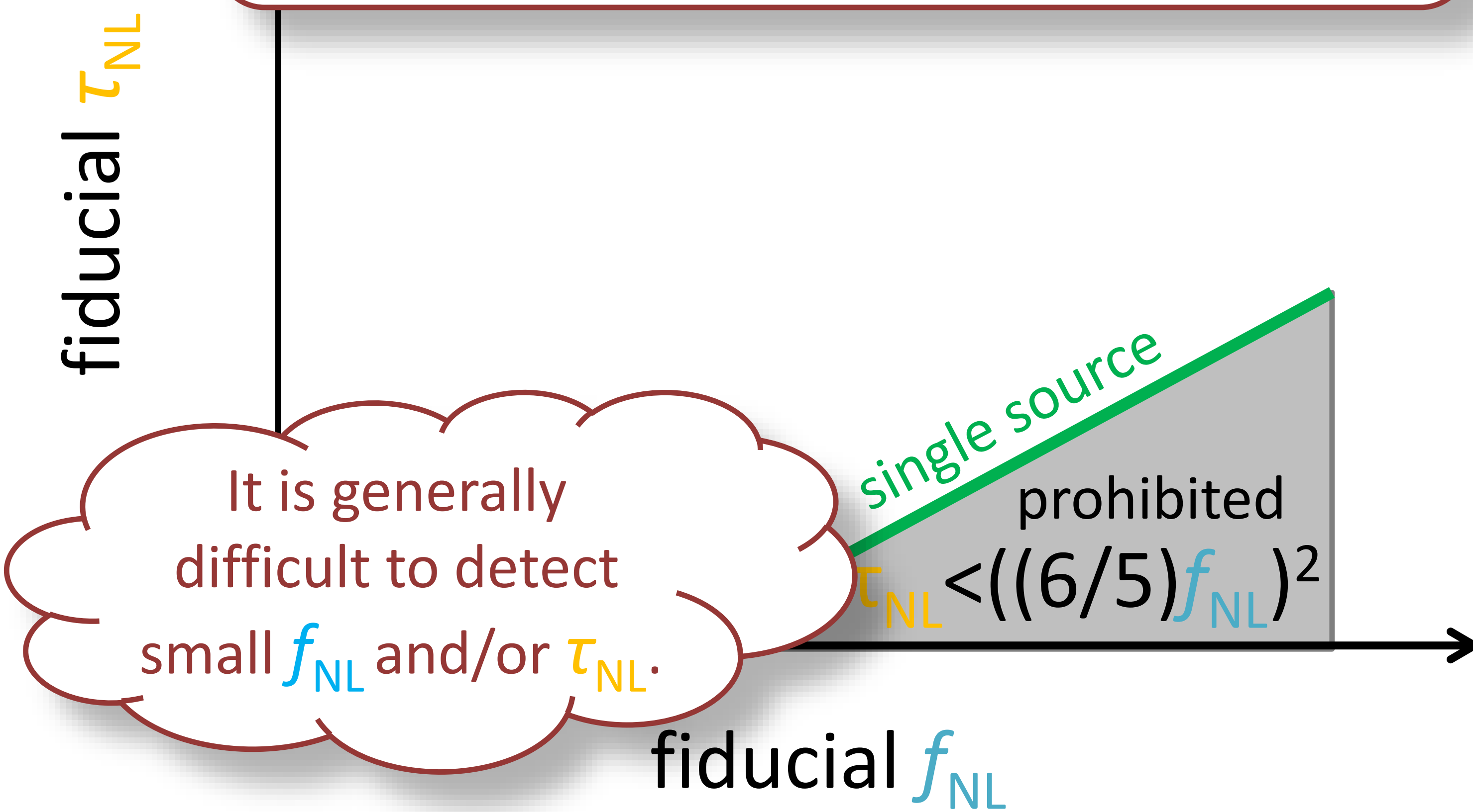
$\sigma(\tau_{NL})$=
43 [SKA1]
23 [SKA2]
62 [Euclid]
(2800 [Planck])

Can we confirm SY-inequality?



Can we confirm SY-inequality?

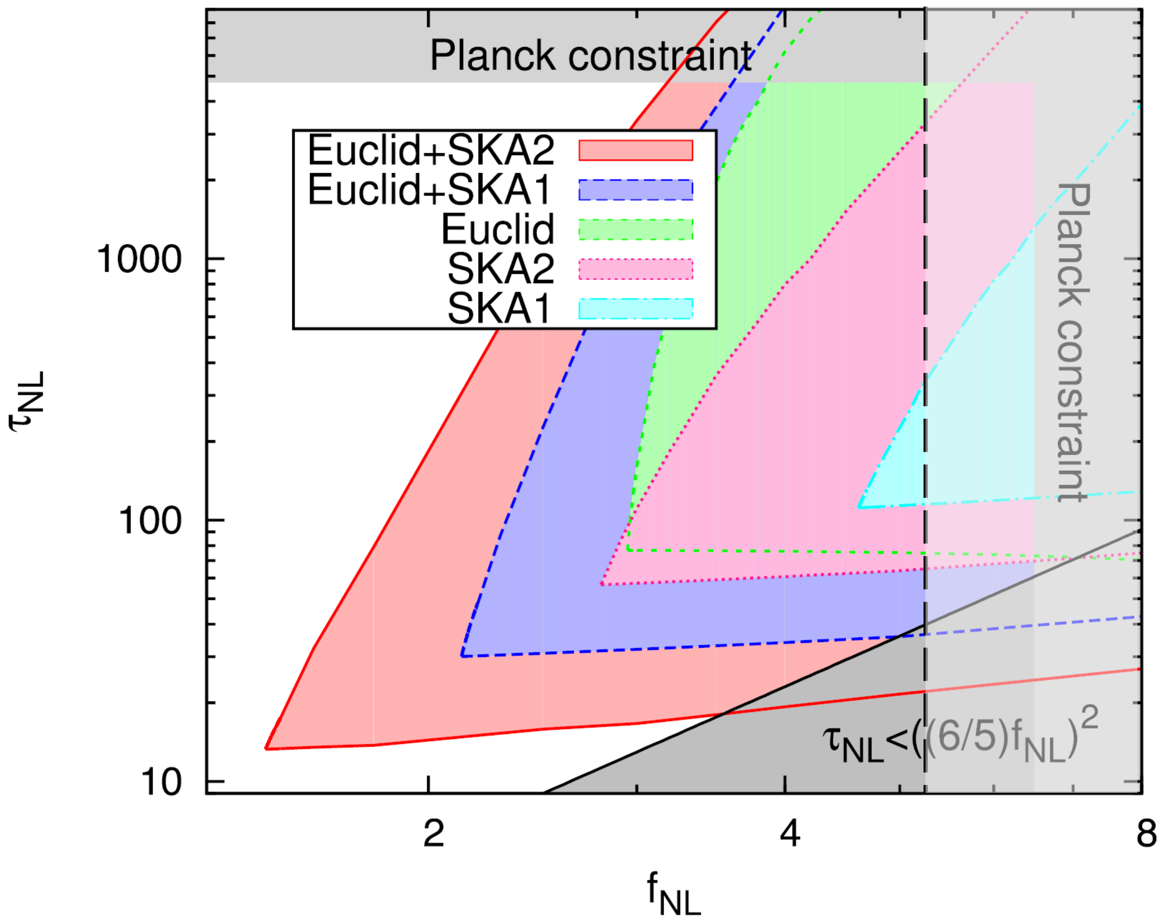
For large τ_{NL}, the constraining power on f_{NL} decreases, because the correction from τ_{NL} to the bias dominates.



Confirming Suyama-Yamaguchi ineq.

[**DY**+Takahashi(2015)]

The region where **both** f_{NL} and τ_{NL} are detected at 1σ



Can we constrain non-local PNG with galaxy surveys?

Local-type/non-local type PNG

- Local type → Can be determined by scale-dependent bias

$$B_\Phi^{\mathrm{local}}(k_1,k_2,k_3) = 2\, f_{\mathrm{NL}}\, (P_\Phi(k_1)P_\Phi(k_2)+\mathrm{cyc})$$

- Non-local type
  - Equilateral

$$B_\Phi^{\mathrm{eq}}(k_1,k_2,k_3) = 6\, f_{\mathrm{NL}}\, [-(P_\Phi(k_1)P_\Phi(k_2)+\mathrm{cyc}) - 2\,(P_\Phi(k_1)P_\Phi(k_2)\,P_\Phi(k_3))^{2/3} + (P_\Phi^{1/3}(k_1)P_\Phi^{2/3}(k_2)\,P_\Phi(k_3)+5\mathrm{perm})\,]$$

  - Orthogonal

$$B_\Phi^{\mathrm{orth}}(k_1,k_2,k_3) = 6\, f_{\mathrm{NL}}\, [-3(P_\Phi(k_1)P_\Phi(k_2)+\mathrm{cyc}) - 8\,(P_\Phi(k_1)P_\Phi(k_2)\,P_\Phi(k_3))^{2/3} + 3\,(P_\Phi^{1/3}(k_1)P_\Phi^{2/3}(k_2)\,P_\Phi(k_3)+5\mathrm{perm})\,]$$

Can galaxy survey constrain these?

non-local type PNG is NOT sensitive to scale-dependent bias

$f_{NL}^{loc} \rightarrow \Delta b[f_{NL}^{loc}] \propto 1/k^2$

$g_{NL}^{loc} \rightarrow \Delta b[g_{NL}^{loc}] \propto 1/k^2$

$\tau_{NL}^{loc} \rightarrow \Delta b[\tau_{NL}^{loc}] \propto 1/k^4$

} Strong low-k dependence

→ Scale-dependent bias in galaxy power spectrum

$f_{NL}^{eq} \rightarrow \Delta b[f_{NL}^{eq}] \propto 1/k^0$

$f_{NL}^{fol} \rightarrow \Delta b[f_{NL}^{fol}] \propto 1/k^1$

$f_{NL}^{orth} \rightarrow \Delta b[f_{NL}^{orth}] \propto 1/k^1$

} Relatively weak

→ Galaxy bispectrum

[e.g., Matsubara(2012)]

Galaxy bispectrum

- Galaxy power spectrum for tracer (a) and (b)

$$P_s^{(a)(b)} = (b_1^{(a)} + f\mu_1^2)\,(b_1^{(b)} + f\mu_2^2)\,P_L(k)$$

- Galaxy bispectrum for tracer (a), (b), and (c)

$$B_{s,grav}^{(a)(b)(c)} = (1/6)[(b_1^{(a)} + f\mu_1^2)(b_1^{(b)} + f\mu_2^2) \times (b_2^{(c)} + 2b_1^{(c)} F_2(\mathbf{k}_1,\mathbf{k}_2)) + (\text{perm})\,]P_L(k_1)P(k_2) + \text{cyc.}$$

Nonlinear bias

↓

includes PNG contribution

2nd order kernel

MT for galaxy bispectrum

- For simplicity, let us consider equilateral bispectrum

$$B^{(abc)}(k,k,k) = \left(b_1^{(a)} b_1^{(b)} b_2^{(c)} + (\mathrm{perm}) \right) P_L^2(k)$$

⇓

$$\sigma(\theta) = (F_{\theta\theta})^{-1/2} = \left(d\boldsymbol{B}/d\theta \cdot \mathrm{Cov}[\boldsymbol{B},\boldsymbol{B}]^{-1} \cdot d\boldsymbol{B}/d\theta \right)^{-1/2}$$

$$\boldsymbol{B} = \{ B^{(111)}, B^{(112)}, B^{(122)}, B^{(222)} \}$$
$$= \{ \alpha^2\gamma B_2, (\alpha^2+2\alpha\gamma)B_2/3, (2\alpha+\gamma)B_2/3, B_2 \}$$

[**DY**+Yokoyama+Takahashi(2017)]

MT for galaxy bispectrum

- For simplicity, let us consider equilateral bispectrum

$$B^{(abc)}(k,k,k) = \left(b_1^{(a)} b_1^{(b)} b_2^{(c)} + (\mathrm{perm}) \right) P_L^2(k)$$

$$\sigma(\gamma) = (F_{\gamma\gamma})^{-1/2} = \left(dB/d\gamma \cdot \mathrm{Cov}[B,B]^{-1} \cdot dB/d\gamma \right)^{-1/2}$$

$\sigma(\gamma = b_2^{(1)}/b_2^{(2)})$

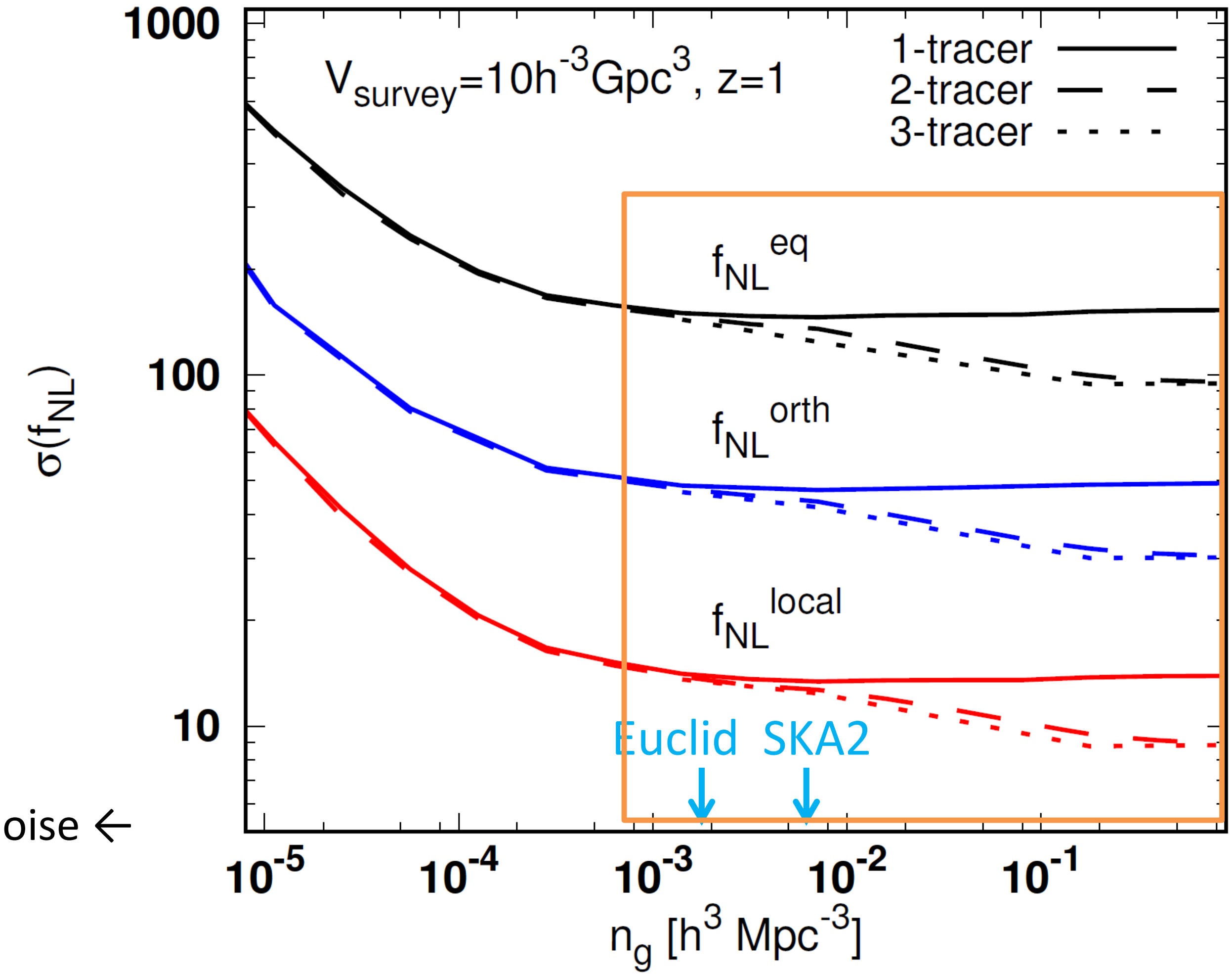
$\rightarrow (3P_2^3/B_2^2)^{1/2} \left((P_2 N_1)^{-1} + \alpha^2 (P_2 N_2)^{-1} \right)^{1/2}$

Ratio of nonlinear bias factor is also determined only by shot noise!

[DY+Yokoyama+Takahashi(2017)]

Error on f_{NL} from galaxy bispectrum

MT is **effective** for low shot-noise region.

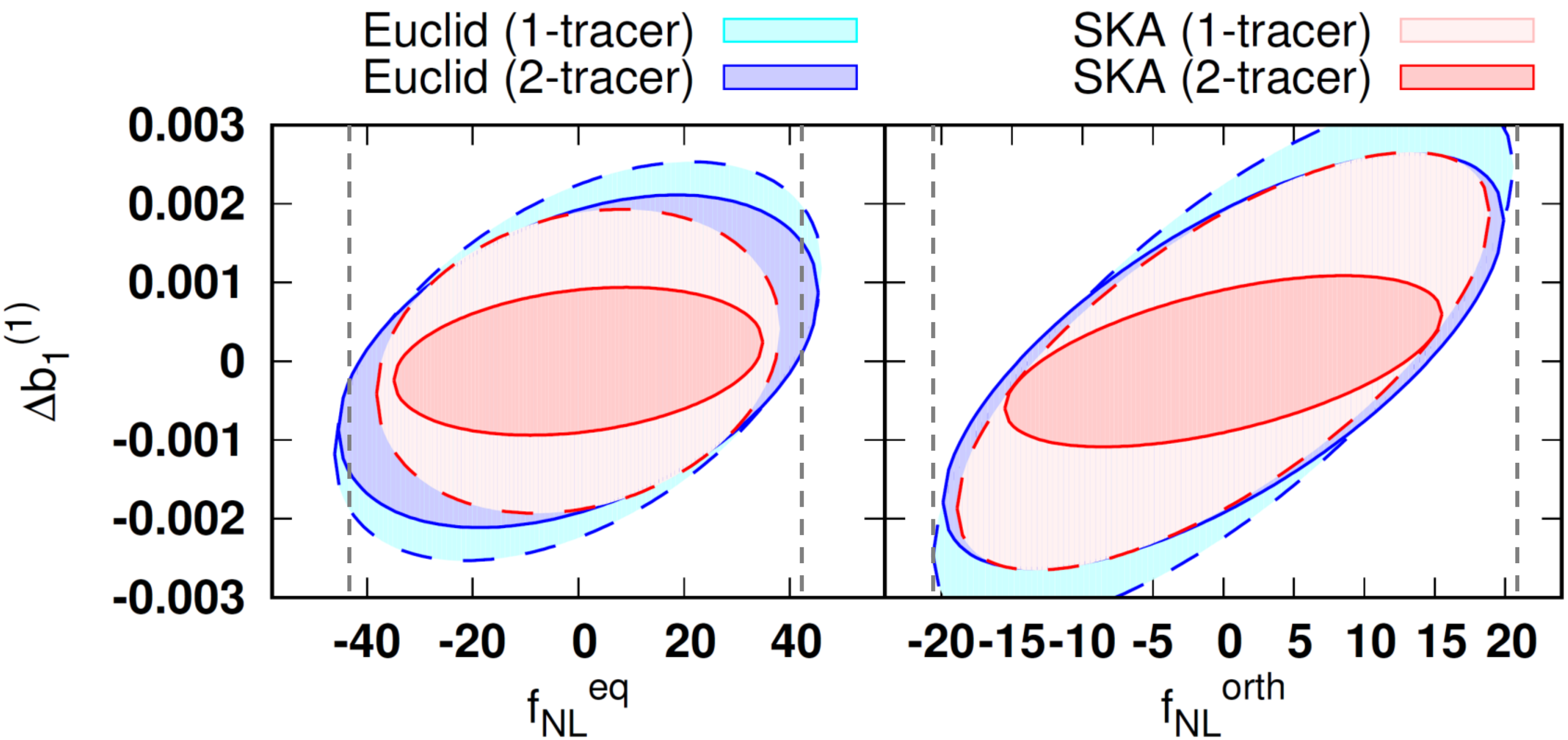


High noise ←

→ Low noise

[**DY**+Yokoyama+Takahashi(2017)]

non-local f_{NL} with galaxy bispectrum



	1-tracer	2-tracer
Planck	(43)	(43)
Euclid	30.4	**30.0**
SKA2	25.1	**23.0**

	1-tracer	2-tracer
Planck	(24)	(24)
Euclid	13.6	**13.1**
SKA2	12.4	**10.2**

[**DY**+S.Yokoyama+T.Takahashi, work in progress]

Scale-dependent PNG

- Genelarized local-type bispectrum [Shandere+Dalal+Huterer (2011)]

$$B_\Phi(k_1,k_2,k_3) = 2\, f_{\rm NL}^{\rm eff}\, [\xi_s(k_3)\xi_m(k_1)\xi_m(k_2)P_\Phi(k_1)P_\Phi(k_2)+\text{cyc.}\,]$$

with $\xi_{s,m}(k)=(k/k_{\rm piv})^{n_{f_{\rm NL}}^{(s,m)}}$

✓ Galaxy bispectrum is useful to break the degeneracy!

✓ Scale-dependent **linear** bias is also induced.

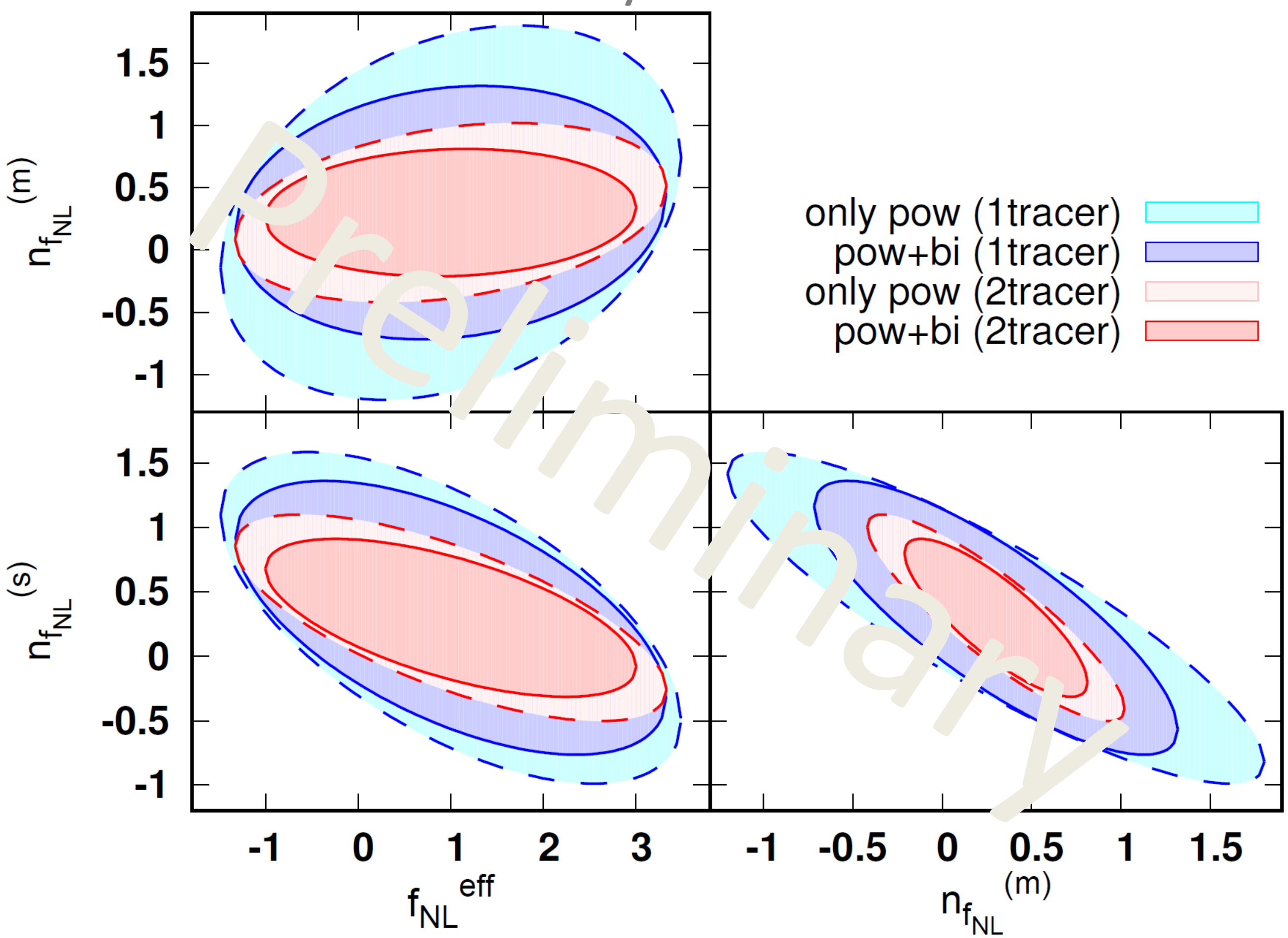
[Matsubara(2012), Desjacques+ (2011),Shandere+ (2011)]

→ Combined analysis of galaxy power- and bi-spectrum

[DY+S.Yokoyama+T.Takahashi, work in progress]

Constraining scale-dependent PNG

SKA2-like survey



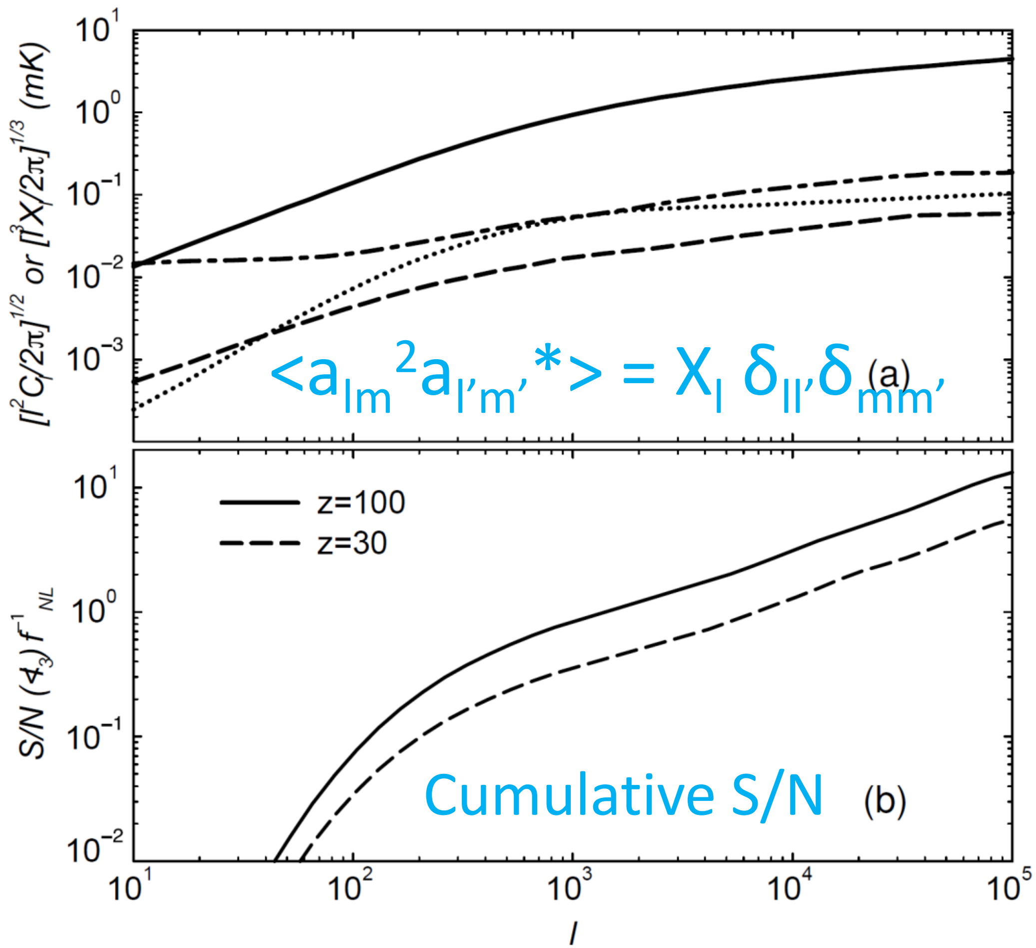
Can we reach f_{NL}~0.01?

Far future 21cm line survey

[Cooray(2006)]

If we use multi-freq information between *z*=100 and *z*=30, we can probe (S/N) = 100 $f_{\rm NL}$

→ fNL = 0.01!



Short summary

- Primordial non-Gaussianity can be constrained by using future LSS surveys!

  - ✓ **Local-type PNG** → galaxy **power** spectrum with scale-dependent bias

    - We may confirm (or reject) SY-inequality in near future(?).

  - ✓ **Non-local PNG** → galaxy **bi**spectrum

What's next?