2014/01/30 Discussion@U Chicago

Derivative interactions in nonlinear massive gravity

YAMAUCHI, Daisuke Research Center for the Early Universe (RESCEU), U. Tokyo

References

Derivative interactions in de Rham-Gabadze-Tolley massive gravity Rampei Kimura(RESCEU->New York U) and DY, PRD88, 084025 (2013), arXiv:1308.0523.

Motivation

Can we construct healthy massive gravity even in nonlinear level?

"Linear" massive gravity

Fierz-Pauli massive gravity [Fierz+Pauli (1939)]

 $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\rm Pl}$ Einstein-Hilbert term $S_{\rm FP} = \int d^4x \left[-\frac{1}{4} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) + \frac{1}{M_{\rm Pl}} h_{\mu\nu} T^{\mu\nu} \right]$

> Only allowed mass term which does not have the ghost at linear order

- (1) Lorentz invariant
- (2) No ghost at linear order
- (3) Simple "nonlinear extension" contains a ghost at nonlinear level [Boulware+Deser(1971)]

Ghost-free nonlinear massive gravity

> Nonlinear extension of Fierz-Pauli massive gravity

[de Rham+Gabadadze+Tolley (2011)]

$$S_{\rm MG} = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left[R - \frac{m^2}{4} \left(\mathcal{U}_2 + \alpha_3 \mathcal{U}_3 + \alpha_4 \mathcal{U}_4 \right) \right] + S_m[g_{\mu\nu}, \psi]$$
$$\mathcal{U}_2 = 2\varepsilon_{\mu\alpha\rho\sigma} \varepsilon^{\nu\beta\rho\sigma} \mathcal{K}^{\mu}_{\ \nu} \mathcal{K}^{\alpha}_{\ \beta}$$
$$\mathcal{U}_3 = \varepsilon_{\mu\alpha\gamma\rho} \varepsilon^{\nu\beta\delta\rho} \mathcal{K}^{\mu}_{\ \nu} \mathcal{K}^{\alpha}_{\ \beta} \mathcal{K}^{\gamma}_{\ \delta}$$
$$\mathcal{U}_4 = \varepsilon_{\mu\alpha\gamma\rho} \varepsilon^{\nu\beta\delta\sigma} \mathcal{K}^{\mu}_{\ \nu} \mathcal{K}^{\alpha}_{\ \beta} \mathcal{K}^{\gamma}_{\ \delta} \mathcal{K}^{\rho}_{\ \sigma}$$
$$\mathcal{K}^{\mu}_{\ \nu} = \delta^{\mu}_{\ \nu} - \sqrt{\delta^{\mu}_{\ \nu} - H^{\mu}_{\ \nu}}$$
$$= \delta^{\mu}_{\ \nu} - \sqrt{\eta_{ab} g^{\mu\alpha} \partial_{\alpha} \phi^a \partial_{\nu} \phi^b}$$

 ϕ^a is called Stuckelberg field, which restores general covariance.

Interactions for linear theories

Fierz-Pauli mass term

$$\mathcal{U}_{\rm FP} = \epsilon^{\mu\alpha\rho\sigma} \epsilon^{\nu\beta}{}_{\rho\sigma} h_{\mu\nu} h_{\alpha\beta}$$

✓ Levi-Civita structure ensures that the Lagrangian is linear in h_{00} .

 \rightarrow h₀₀ becomes a Lagrange multiplier, which kills the ghost!

Derivative interaction in FP theory

[Hinterbichler(2013)] [see also Folkerts+Pritzel+Wintergerst(2011)]

$$\mathcal{L}_{2,3} \sim M_{\rm Pl}^2 \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} \partial_{\mu} \partial_{\alpha} h_{\nu\beta} h_{\rho\gamma} h_{\sigma\delta}$$

 \rightarrow h₀₀ becomes a Lagrange multiplier.

Our work

Is there any consistent "nonlinear extension" of derivative interactions in de Rham-Gabadadze-Tolley massive gravity?

Guideline for construction of Lagrangian

To construct "nonlinear" derivative interaction in dRGT massive gravity, we demand the following restrictions:

(1) Linearization of $h_{\mu\nu}$ reproduces Fierz-Pauli massive gravity.

- ✓ Lorentz invariance
- ✓ Free of Boulware-Deser ghost at linear order
- (2) The cutoff energy scale is Λ_3 .

✓ All nonlinear terms below Λ_3 have to be eliminated.

(3) The resultant theory does not have a BD ghost.

Candidates for derivative interactions

There are a number of candidates for nonlinear interactions, such as

$$\mathcal{L}_{\text{int}} \supset M_{\text{Pl}}^2 \sqrt{-g} HR, \ M_{\text{Pl}}^2 \sqrt{-g} H^2R, \ M_{\text{Pl}}^2 \sqrt{-g} H^3R, \ \cdots$$

✓ Define the tensor $H_{\mu\nu}$ as the covariantization of the metric perturbations as

$$H_{\mu\nu} = g_{\mu\nu} - \eta_{ab} \partial_{\mu} \phi^a \partial_{\nu} \phi^b$$

where the four Stuckelberg fields ϕ^a transform as scalars.

 Note : We restrict the form of derivative interactions by using only a Riemann tensor.

Decoupling limit (DL)

We can safely ignore vector modes in DL and the helocity-0 mode π of the graviton can be extracted by expressing

$$\phi^a = \delta^a_\mu x^\mu - \eta^{a\mu} \partial_\mu \pi / M_{\rm Pl} m^2$$

DL is convenient to capture high energy behavior below the Compton wavelength of massive graviton:

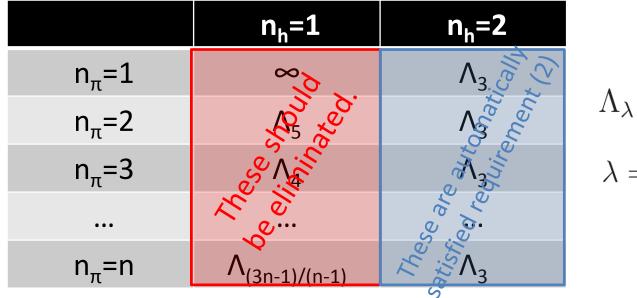
$$M_{\rm Pl} \to \infty, \ m \to 0, \ \Lambda_3 = (M_{\rm Pl}m^2)^{1/3} = \text{fixed},$$

 $H_{\mu\nu} \to \frac{h_{\mu\nu}}{M_{\rm Pl}} + 2\frac{\partial_{\mu}\partial_{\nu}\pi}{M_{\rm Pl}m^2} - \frac{\partial_{\mu}\partial_{\alpha}\pi\partial_{\nu}\partial^{\alpha}\pi}{M_{\rm Pl}^2m^4}$

Energy scale for derivative interactions

The Lagrangian in DL can be schematically written as

$$\mathcal{L}_{\text{int}} \sim \Lambda_{\lambda}^{2-n_h-3n_{\pi}} h^{n_h-1} \partial^2 h \, (\partial^2 \pi)^{n_{\pi}}$$



$$\Lambda_{\lambda} = (M_{\rm Pl}m^{\lambda-1})^{1/\lambda}$$
$$\lambda = \frac{n_h + 3n_\pi - 2}{n_h + n_\pi - 2}$$

The energy scale of n_{π} =1 terms in the DL is potentially dangerous and these terms have to be eliminated (requirement (2)).

HR order

We start with the lowest order terms in a general form;

$$\mathcal{L}_{\text{int},1} = M_{\text{Pl}}^2 \sqrt{-g} H_{\mu\nu} (R^{\mu\nu} + d R g^{\mu\nu})$$

To satisfy the requirement (1) : Linearization of $h_{\mu\nu}$ reproduces the FP theory, we require d=-1/2. In terms of Levi-Civita symbol,

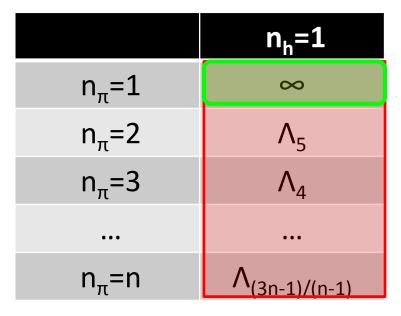
$$\mathcal{L}_{\text{int},1} = \frac{1}{2} M_{\text{Pl}}^2 \sqrt{-g} \, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} R_{\mu\alpha\nu\beta} \, H_{\rho\gamma}$$

HR order in DL

✓ The lowest order term in DL, $(n_h, n_\pi)=(1, 1)$, is given by

$$\mathcal{L}_{\text{int},1}\Big|_{\partial^2 h \,\partial^2 \pi} = -\frac{1}{m^2} \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} \partial_{\mu} \partial_{\alpha} h_{\nu\beta} \partial_{\rho} \partial_{\gamma} \pi$$
$$= -\frac{1}{m^2} \partial_{\gamma} (\varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} \partial_{\mu} \partial_{\alpha} h_{\nu\beta} \partial_{\rho} \pi)$$

Total derivative!

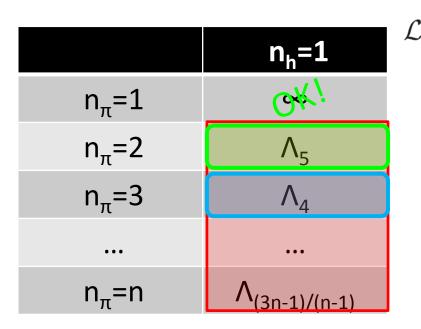


HR order in DL

✓ The next order term in DL, $(n_h, n_\pi)=(1, 2)$, is not total derivative:

$$\mathcal{L}_{\text{int},1}\Big|_{\partial^2 h\,(\partial^2 \pi)^2} = \frac{1}{2\Lambda_5^5} \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} \partial_{\mu} \partial_{\alpha} h_{\nu\beta} \partial_{\rho} \partial_{\kappa} \pi \partial^{\kappa} \partial_{\gamma} \pi$$

 \rightarrow Only way to eliminate this term is to add the next order term:



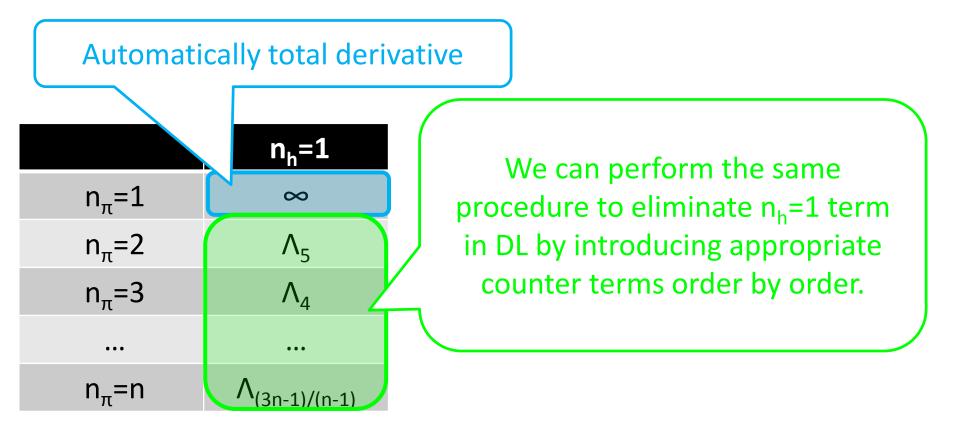
$$\mathcal{L}_{\text{int},1,2} = \frac{1}{8} M_{\text{Pl}}^2 \sqrt{-g} \, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}_{\sigma} R_{\mu\alpha\nu\beta} \, H_{\rho\kappa} H^{\kappa}_{\gamma}$$
$$\boldsymbol{\Lambda}_{5} \text{ term can be eliminated!}$$

...but it contains n_{π} =3 term which is not total derivative:

$$\frac{1}{2\Lambda_4^8} \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} \partial_{\mu} \partial_{\alpha} h_{\nu\beta} \partial_{\rho} \partial_{\kappa} \pi \partial^{\kappa} \partial_{\lambda} \pi \partial^{\lambda} \partial_{\gamma} \pi$$

HR order in DL

✓ Following the same step as $n_{\pi}=2$, the $n_{\pi}=3$ dangerous term can be eliminated by adding higher-order terms. Then...



Total HR order Lagrangian

✓ The total Lagrangian including counter terms is given by

$$\mathcal{L}_{\text{int},1} = M_{\text{Pl}}^2 \sqrt{-g} \, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} R_{\mu\alpha\nu\beta} \\ \times \left(\frac{1}{2} H_{\rho\gamma} + \frac{1}{8} H_{\rho\kappa} H^{\kappa}{}_{\gamma} + \frac{1}{16} H_{\rho\kappa} H^{\kappa}{}_{\lambda} H^{\lambda}{}_{\gamma} + \frac{5}{128} H_{\rho\kappa} H^{\kappa}{}_{\lambda} H^{\lambda}{}_{\tau} H^{\tau}{}_{\gamma} + \cdots \right) \\ = \mathcal{K}_{\rho\gamma} \\ \mathcal{L}_{\text{int},1} = M_{\text{Pl}}^2 \sqrt{-g} \, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} R_{\mu\alpha\nu\beta} \, \mathcal{K}_{\rho\gamma}$$

- This Lagrangian does not have the terms of the energy scales bellow Λ₃, and nonlinear terms contribute at Λ₃.
- We have only on n_h=1 term in DL, and K tensor ensures n_h=1 term to be a total derivative in DL.

H²R order

 \checkmark The starting point of the Lagrangian is given by

$$\mathcal{L}_{\text{int},2} = \frac{1}{4} M_{\text{Pl}}^2 \sqrt{-g} \,\varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} R_{\mu\alpha\nu\beta} \,H_{\rho\gamma} \,H_{\sigma\delta}$$

This is the only combination that the lowest order Λ_5 term becomes a total derivative.

✓ With the same procedure of the previous case, the counter terms can be resumed by using K tensor again,

$$\mathcal{L}_{\text{int},2} = M_{\text{Pl}}^2 \sqrt{-g} \, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} R_{\mu\alpha\nu\beta} \, \mathcal{K}_{\rho\gamma} \, \mathcal{K}_{\sigma\delta}$$

Note : Linearization of $h_{\mu\nu}$ in unitary gauge gives the "pseudolinear" derivative interaction derived by Hinterbichler (2013).

H³R or higher order

We do not have total derivative combination due to the number of indices of antisymmetric tensor in 4 dim.



There is no higher order Lagrangian satisfying the restrictions.

✓ The derivative interaction for massive gravity we found is

$$\mathcal{L}_{\rm int} = M_{\rm Pl}^2 \sqrt{-g} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} R_{\mu\alpha\nu\beta} \left(\alpha g_{\rho\gamma} \mathcal{K}_{\sigma\delta} + \beta \mathcal{K}_{\rho\gamma} \mathcal{K}_{\sigma\delta} \right)$$

BD ghost???

So far we could successfully eliminate all energy scale below Λ_3 , and the derivative interaction in DL contribute at Λ_3 . But we still need to check the requirement (3) : BD ghost.

$$\searrow \Lambda_3 \text{ theory in DL } (n_h=2, n_\pi=\text{arbitrary})$$

$$\mathcal{L} \sim \frac{1}{\Lambda_3^3} \pi \Big[R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \Big] + \frac{1}{\Lambda_3^{3n_\pi}} \mathcal{O} \Big[h\partial^2 h \left(\partial^2 \pi \right)^{n_\pi} \Big]$$
EoM is 2nd order differential equation
These terms yield 4th order
differential equations for h and π ...

There appear extra degrees of freedom, which leads to a ghost... The mass scale of the derivative interactions should be $M < M_{pl}$.

Summary

We found the nonlinear derivative interaction in dRGT massive gravity.

- ✓ The energy scales below Λ_3 can be eliminated by adding counter terms.
- ✓ The Lagrangian can be resumed by using K tensor.
- ✓ Nonlinear terms contribute at Λ_3 .
- ✓ There appear fourth order differential equations of the scalar and tensor modes in DL, which leads to a ghost....

[see also de Rham+Matas+Tolley (2013)]

Other candidates (in progress...)

We found other A3 derivative interactions without the Riemann tensor. There exist other derivative interactions using the covariant derivative of K tensor.

$$\mathcal{L}_{2}^{(2)} = \frac{1}{2} M^{2} \sqrt{|g|} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} \nabla_{\mu} \mathcal{K}_{\nu\beta} \nabla_{\alpha} \mathcal{K}_{\rho\gamma} g_{\sigma\delta} ,$$

$$\mathcal{L}_{3}^{(2)} = M^{2} \sqrt{|g|} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} \nabla_{\mu} \mathcal{K}_{\nu\beta} \nabla_{\alpha} \mathcal{K}_{\rho\gamma} F_{3,\sigma\delta} (H)$$

But ... so far we cannot kill higher derivative terms in EoM even if we combine all possible derivative interaction terms.