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Derivative interactions in nonlinear massive gravity

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References

Derivative interactions in de Rham-Gabadze-Tolley massive gravity
Rampei Kimura(RESCEU->New York U) and **DY**,
PRD88, 084025 (2013), arXiv:1308.0523.

Motivation

Can we construct healthy massive gravity even in nonlinear level?

"Linear" massive gravity

- Fierz-Pauli massive gravity [Fierz+Pauli (1939)]

Linearized Einstein-Hilbert term

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\rm Pl}$$

$$S_{\rm FP} = \int d^4x \left[-\frac{1}{4} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) + \frac{1}{M_{\rm Pl}} h_{\mu\nu} T^{\mu\nu} \right]$$

Only allowed mass term which does not have the ghost at linear order

(1) Lorentz invariant
(2) No ghost at linear order
(3) Simple "nonlinear extension" contains a ghost at nonlinear level
[Boulware+Deser(1971)]

Ghost-free nonlinear massive gravity

➢ Nonlinear extension of Fierz-Pauli massive gravity

[de Rham+Gabadadze+Tolley (2011)]

$$S_{\rm MG} = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}\left(\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4\right)\right] + S_m[g_{\mu\nu},\psi]$$

$$\begin{aligned}
\mathcal{U}_2 &= 2\varepsilon_{\mu\alpha\rho\sigma}\varepsilon^{\nu\beta\rho\sigma}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta \\
\mathcal{U}_3 &= \varepsilon_{\mu\alpha\gamma\rho}\varepsilon^{\nu\beta\delta\rho}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta\mathcal{K}^\gamma{}_\delta \\
\mathcal{U}_4 &= \varepsilon_{\mu\alpha\gamma\rho}\varepsilon^{\nu\beta\delta\sigma}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta\mathcal{K}^\gamma{}_\delta\mathcal{K}^\rho{}_\sigma
\end{aligned}$$

$$\begin{aligned}
\mathcal{K}^\mu{}_\nu &= \delta^\mu{}_\nu - \sqrt{\delta^\mu{}_\nu - H^\mu{}_\nu} \\
&= \delta^\mu{}_\nu - \sqrt{\eta_{ab}g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b}
\end{aligned}$$

ϕ^a is called Stuckelberg field, which restores general covariance.

Interactions for linear theories

- Fierz-Pauli mass term

$$\mathcal{U}_{\rm FP} = \epsilon^{\mu\alpha\rho\sigma}\epsilon^{\nu\beta}{}_{\rho\sigma}h_{\mu\nu}h_{\alpha\beta}$$

  - ✓ Levi-Civita structure ensures that the Lagrangian is linear in h_{00}.

    → h_{00} becomes a Lagrange multiplier, which kills the ghost!

- Derivative interaction in FP theory

  [Hinterbichler(2013)] [see also Folkerts+Pritzel+Wintergerst(2011)]

$$\mathcal{L}_{2,3} \sim M_{\rm Pl}^2 \varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma\delta}\partial_\mu\partial_\alpha h_{\nu\beta}h_{\rho\gamma}h_{\sigma\delta}$$

→ h_{00} becomes a Lagrange multiplier.

Our work

Is there any consistent "nonlinear extension"
of derivative interactions
in de Rham-Gabadadze-Tolley massive gravity?

Guideline for construction of Lagrangian

To construct "nonlinear" derivative interaction in dRGT massive gravity, we demand the following restrictions:

(1) Linearization of $h_{\mu\nu}$ reproduces Fierz-Pauli massive gravity.

- ✓ Lorentz invariance
- ✓ Free of Boulware-Deser ghost at linear order

(2) The cutoff energy scale is Λ_3.

- ✓ All nonlinear terms below Λ_3 have to be eliminated.

(3) The resultant theory does not have a BD ghost.

Candidates for derivative interactions

There are a number of candidates for nonlinear interactions, such as

$$\mathcal{L}_{\rm int} \supset M_{\rm Pl}^2\sqrt{-g}HR,\ M_{\rm Pl}^2\sqrt{-g}H^2R,\ M_{\rm Pl}^2\sqrt{-g}H^3R,\ \cdots.$$

✓ Define the tensor $H_{\mu\nu}$ as the covariantization of the metric perturbations as

$$H_{\mu\nu} = g_{\mu\nu} - \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

where the four Stuckelberg fields ϕ^a transform as scalars.

✓ Note : We restrict the form of derivative interactions by using only a Riemann tensor.

Decoupling limit (DL)

- We can safely ignore vector modes in DL and the helocity-0 mode π of the graviton can be extracted by expressing

$$\phi^a = \delta^a_\mu x^\mu - \eta^{a\mu}\partial_\mu \pi / M_{\rm Pl} m^2$$

- DL is convenient to capture high energy behavior below the Compton wavelength of massive graviton:

$$M_{\rm Pl} \to \infty,\ m \to 0,\ \Lambda_3 = (M_{\rm Pl} m^2)^{1/3} = \text{fixed},$$

$$H_{\mu\nu} \to \frac{h_{\mu\nu}}{M_{\rm Pl}} + 2\frac{\partial_\mu\partial_\nu\pi}{M_{\rm Pl}m^2} - \frac{\partial_\mu\partial_\alpha\pi\partial_\nu\partial^\alpha\pi}{M_{\rm Pl}^2 m^4}$$

Energy scale for derivative interactions

The Lagrangian in DL can be schematically written as

$$\mathcal{L}_{\rm int} \sim \Lambda_\lambda^{2-n_h-3n_\pi} h^{n_h-1}\partial^2 h\,(\partial^2\pi)^{n_\pi}$$

	n_h=1	n_h=2
n_π=1	∞	Λ_3
n_π=2	Λ_5	Λ_3
n_π=3	Λ_4	Λ_3
…	…	…
n_π=n	$\Lambda_{(3n-1)/(n-1)}$	Λ_3

These should be eliminated.

These are automatically satisfied requirement (2)

$$\Lambda_\lambda = (M_{\rm Pl} m^{\lambda-1})^{1/\lambda}$$

$$\lambda = \frac{n_h+3n_\pi-2}{n_h+n_\pi-2}$$

The energy scale of n_π=1 terms in the DL is potentially dangerous and these terms have to be eliminated (requirement (2)).

HR order

We start with the lowest order terms in a general form;

$$\mathcal{L}_{\rm int,1} = M_{\rm Pl}^2 \sqrt{-g} H_{\mu\nu} (R^{\mu\nu} + d\, R g^{\mu\nu})$$

To satisfy the requirement (1) : Linearization of $h_{\mu\nu}$ reproduces the FP theory, we require *d=-1/2*. In terms of Levi-Civita symbol,

$$\mathcal{L}_{\rm int,1} = \frac{1}{2} M_{\rm Pl}^2 \sqrt{-g}\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} R_{\mu\alpha\nu\beta}\, H_{\rho\gamma}$$

HR order in DL

✓ The lowest order term in DL, $(n_h,n_\pi)=(1,1)$, is given by

$$\mathcal{L}_{\rm int,1}\Big|_{\partial^2 h\,\partial^2\pi} = -\frac{1}{m^2}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma}\partial_\mu\partial_\alpha h_{\nu\beta}\,\partial_\rho\partial_\gamma\pi$$

$$= -\frac{1}{m^2}\partial_\gamma\left(\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma}\partial_\mu\partial_\alpha h_{\nu\beta}\,\partial_\rho\pi\right)$$

Total derivative!

	$n_h=1$
$n_\pi=1$	∞
$n_\pi=2$	Λ_5
$n_\pi=3$	Λ_4
…	…
$n_\pi=n$	$\Lambda_{(3n-1)/(n-1)}$

HR order in DL

✓ The next order term in DL, $(n_h, n_\pi)=(1,2)$, **is not total derivative**:

$$\mathcal{L}_{\rm int,1}\Big|_{\partial^2 h\,(\partial^2\pi)^2} = \frac{1}{2\Lambda_5^5}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma}\partial_\mu\partial_\alpha h_{\nu\beta}\,\partial_\rho\partial_\kappa\pi\partial^\kappa\partial_\gamma\pi$$

→Only way to eliminate this term is to add the next order term:

$$\mathcal{L}_{\rm int,1,2} = \frac{1}{8}M_{\rm Pl}^2\sqrt{-g}\,\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma}R_{\mu\alpha\nu\beta}\,H_{\rho\kappa}H^{\kappa}{}_{\gamma}$$

	$n_h=1$
$n_\pi=1$	OK!
$n_\pi=2$	Λ_5
$n_\pi=3$	Λ_4
…	…
$n_\pi=n$	$\Lambda_{(3n-1)/(n-1)}$

Λ_5 term can be eliminated!

…but it contains $n_\pi=3$ term which is not total derivative:

$$\frac{1}{2\Lambda_4^8}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_{\sigma}\partial_\mu\partial_\alpha h_{\nu\beta}\,\partial_\rho\partial_\kappa\pi\partial^\kappa\partial_\lambda\pi\partial^\lambda\partial_\gamma\pi$$

HR order in DL

✓ Following the same step as n_π=2, the n_π=3 dangerous term can be eliminated by adding higher-order terms. Then…

Automatically total derivative

	$n_h=1$
$n_\pi=1$	∞
$n_\pi=2$	Λ_5
$n_\pi=3$	Λ_4
…	…
$n_\pi=n$	$\Lambda_{(3n-1)/(n-1)}$

We can perform the same procedure to eliminate n_h=1 term in DL by introducing appropriate counter terms order by order.

Total HR order Lagrangian

✓ The total Lagrangian including counter terms is given by

$$\mathcal{L}_{\mathrm{int},1} = M_{\mathrm{Pl}}^2 \sqrt{-g}\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} R_{\mu\alpha\nu\beta} \times \left(\frac{1}{2} H_{\rho\gamma} + \frac{1}{8} H_{\rho\kappa} H^{\kappa}{}_{\gamma} + \frac{1}{16} H_{\rho\kappa} H^{\kappa}{}_{\lambda} H^{\lambda}{}_{\gamma} + \frac{5}{128} H_{\rho\kappa} H^{\kappa}{}_{\lambda} H^{\lambda}{}_{\tau} H^{\tau}{}_{\gamma} + \cdots \right)$$

$$= \mathcal{K}_{\rho\gamma}$$

$$\mathcal{L}_{\mathrm{int},1} = M_{\mathrm{Pl}}^2 \sqrt{-g}\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma}{}_{\sigma} R_{\mu\alpha\nu\beta}\, \mathcal{K}_{\rho\gamma}$$

- This Lagrangian does not have the terms of the energy scales bellow Λ_3, and nonlinear terms contribute at Λ_3.
- We have only on n_h=1 term in DL, and K tensor ensures n_h=1 term to be a total derivative in DL.

H^2R order

✓ The starting point of the Lagrangian is given by

$$\mathcal{L}_{\rm int,2} = \frac{1}{4} M_{\rm Pl}^2 \sqrt{-g}\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} R_{\mu\alpha\nu\beta}\, H_{\rho\gamma}\, H_{\sigma\delta}$$

This is the only combination that the lowest order Λ_5 term becomes a total derivative.

✓ With the same procedure of the previous case, the counter terms can be resumed by using K tensor again,

$$\mathcal{L}_{\rm int,2} = M_{\rm Pl}^2 \sqrt{-g}\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} R_{\mu\alpha\nu\beta}\, \mathcal{K}_{\rho\gamma}\, \mathcal{K}_{\sigma\delta}$$

Note : Linearization of $h_{\mu\nu}$ in unitary gauge gives the "pseudo-linear" derivative interaction derived by Hinterbichler (2013).

H^3R or higher order

We do not have total derivative combination due to the number of indices of antisymmetric tensor in 4 dim.

There is no higher order Lagrangian satisfying the restrictions.

✓ The derivative interaction for massive gravity we found is

$$\mathcal{L}_{\rm int} = M_{\rm Pl}^2 \sqrt{-g}\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}R_{\mu\alpha\nu\beta}\left(\alpha g_{\rho\gamma}\mathcal{K}_{\sigma\delta} + \beta\mathcal{K}_{\rho\gamma}\mathcal{K}_{\sigma\delta}\right)$$

BD ghost???

So far we could successfully eliminate all energy scale below Λ_3, and the derivative interaction in DL contribute at Λ_3.
But we still need to check the requirement (3) : BD ghost.

- Λ_3 theory in DL (n_h=2, n_π=arbitrary)

$$\mathcal{L} \sim \frac{1}{\Lambda_3^3}\pi\left[R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right] + \frac{1}{\Lambda_3^{3n_\pi}}\mathcal{O}\left[h\partial^2 h\left(\partial^2\pi\right)^{n_\pi}\right]$$

EoM is 2nd order differential equation

These terms yield 4th order differential equations for h and π…

There appear extra degrees of freedom, which leads to a ghost…
The mass scale of the derivative interactions should be $M<M_{pl}$.

Summary

We found the nonlinear derivative interaction in dRGT massive gravity.

- ✓ The energy scales below Λ_3 can be eliminated by adding counter terms.
- ✓ The Lagrangian can be resumed by using K tensor.
- ✓ Nonlinear terms contribute at Λ_3.
- ✓ There appear fourth order differential equations of the scalar and tensor modes in DL, which leads to a ghost....

[see also de Rham+Matas+Tolley (2013)]

Other candidates (in progress…)

We found other Λ3 derivative interactions without the Riemann tensor. There exist other derivative interactions using the covariant derivative of K tensor.

$$\mathcal{L}_2^{(2)} = \frac{1}{2} M^2 \sqrt{|g|} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} \nabla_\mu \mathcal{K}_{\nu\beta} \nabla_\alpha \mathcal{K}_{\rho\gamma} g_{\sigma\delta} \, ,$$

$$\mathcal{L}_3^{(2)} = M^2 \sqrt{|g|} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} \nabla_\mu \mathcal{K}_{\nu\beta} \nabla_\alpha \mathcal{K}_{\rho\gamma} F_{3,\sigma\delta} (H)$$

But … so far we cannot kill higher derivative terms in EoM even if we combine all possible derivative interaction terms.