2014/01/30 KICP seminar, U Chicago

Full-sky formulae for weak lensing power spectra from total angular momentum method

YAMAUCHI, Daisuke

Research Center for the Early Universe (RESCEU), U. Tokyo

References

Weak lensing generated by vector perturbations and detectability of cosmic strings
DY, A. Taruya, T. Namikawa, JCAP08(2012)030, arXiv:1205.2139.

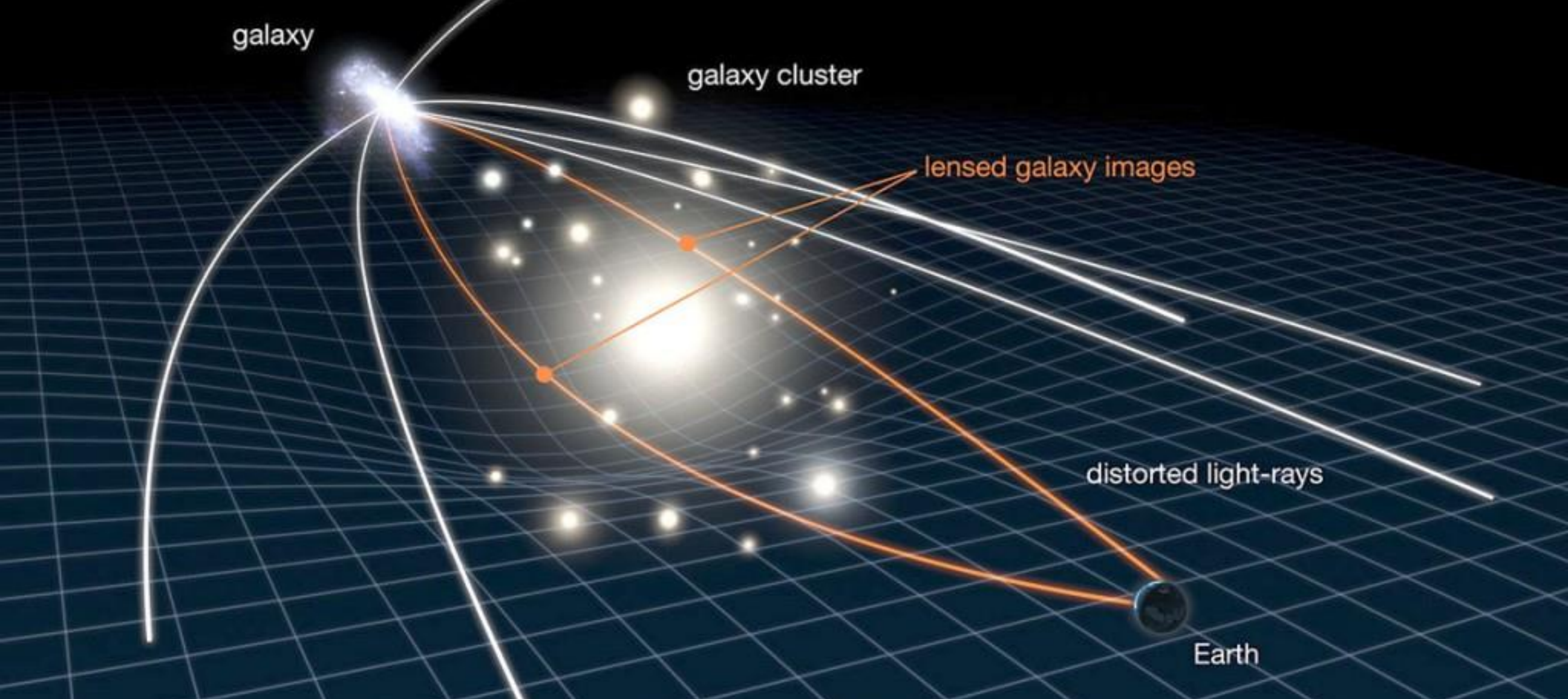
Full-sky formulae for weak lensing power spectra from total angular momentum method
DY, A. Taruya, T. Namikawa, JCAP08(2013)051, arXiv:1305.3348.

Constraining cosmic string parameters with curl mode of CMB lensing
T. Namikawa, **DY**, A. Taruya, PRD88, 083525, arXiv:1308.6068.

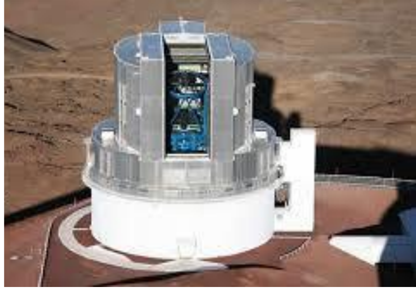
INTRODUCTION

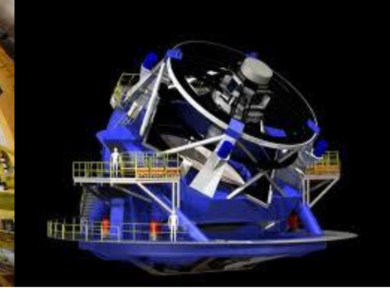
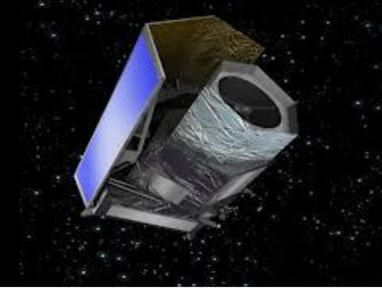
NASA/ESA

Gravitational Lensing
= method to ``see'' invisibles



WEAK LENSING observations can provide a direct evidence for the intervening ``Scalar/Vector/Tensor modes'' along a line of sight by measuring the spatial patterns of the deformation of the photon path.



Imaging survey

e.g. HSC, DES, LSST, Euclid…

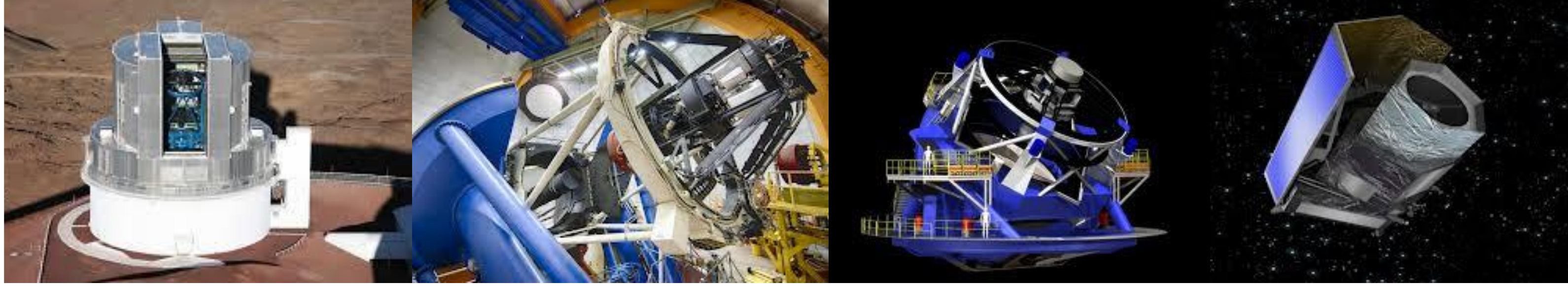
Weak lensing measurement

CMB lensing

e.g. ACT, SPT, Planck, PolarBear, ACTPol, SPTPol, COrE,…







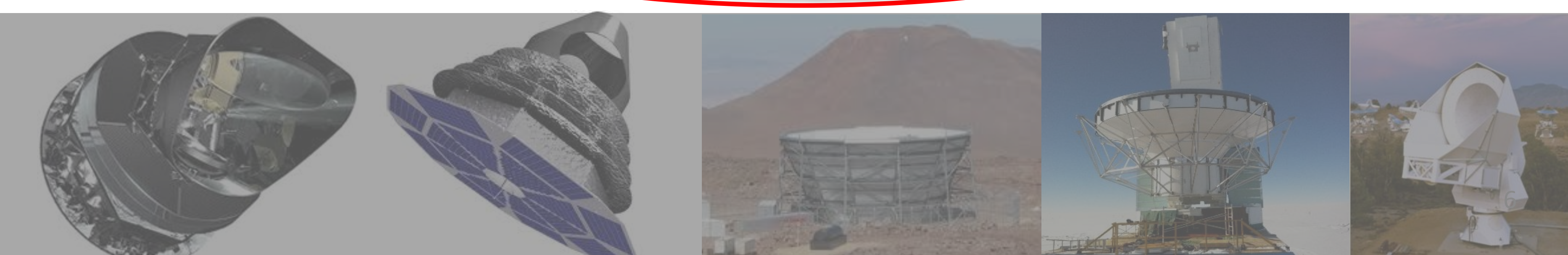
Imaging survey

e.g. HSC, DES, LSST, Euclid…

Weak lensing measurement

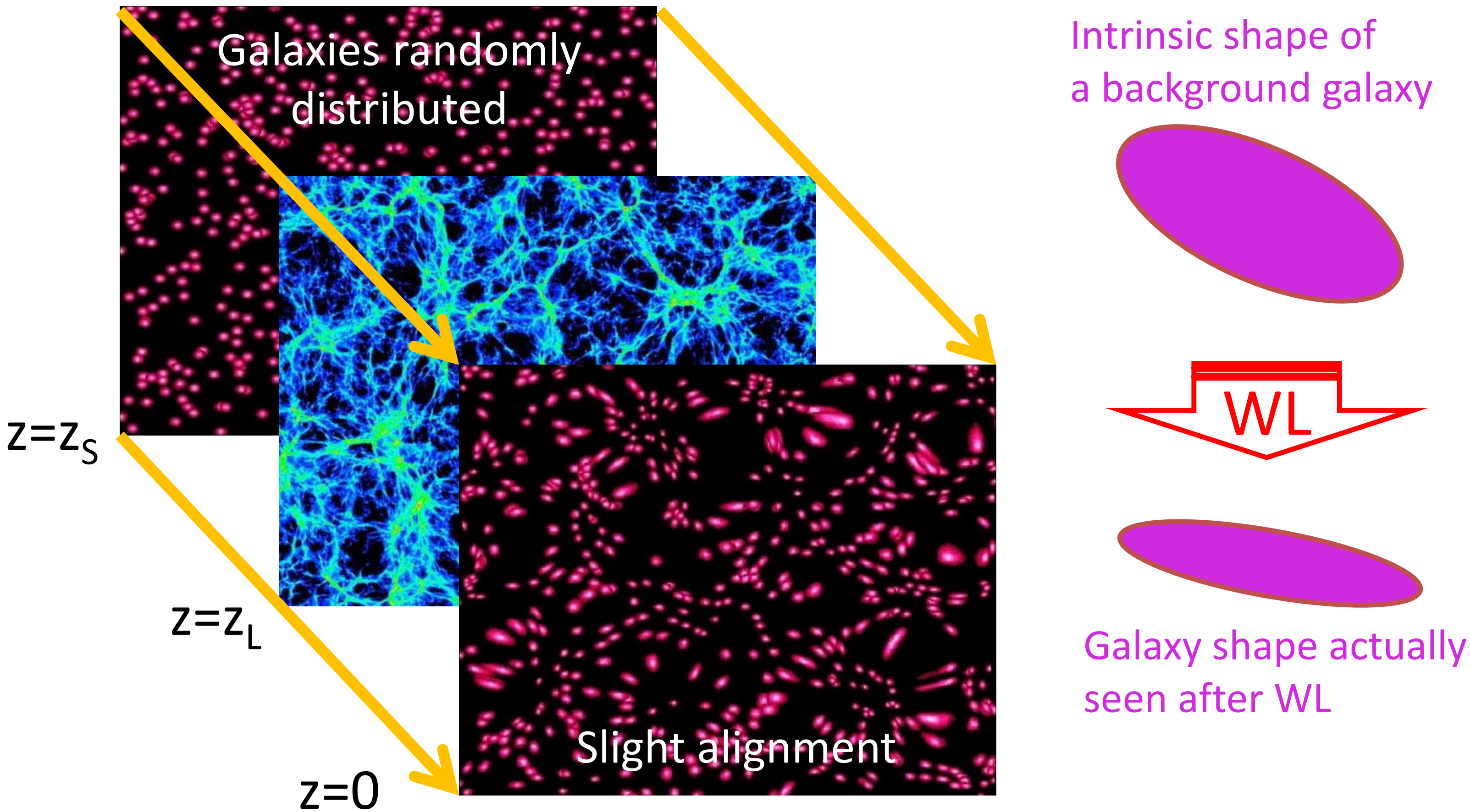
CMB lensing

e.g. ACT, SPT, Planck, PolarBear, ACTPol, SPTPol, COrE,…



Cosmic shear

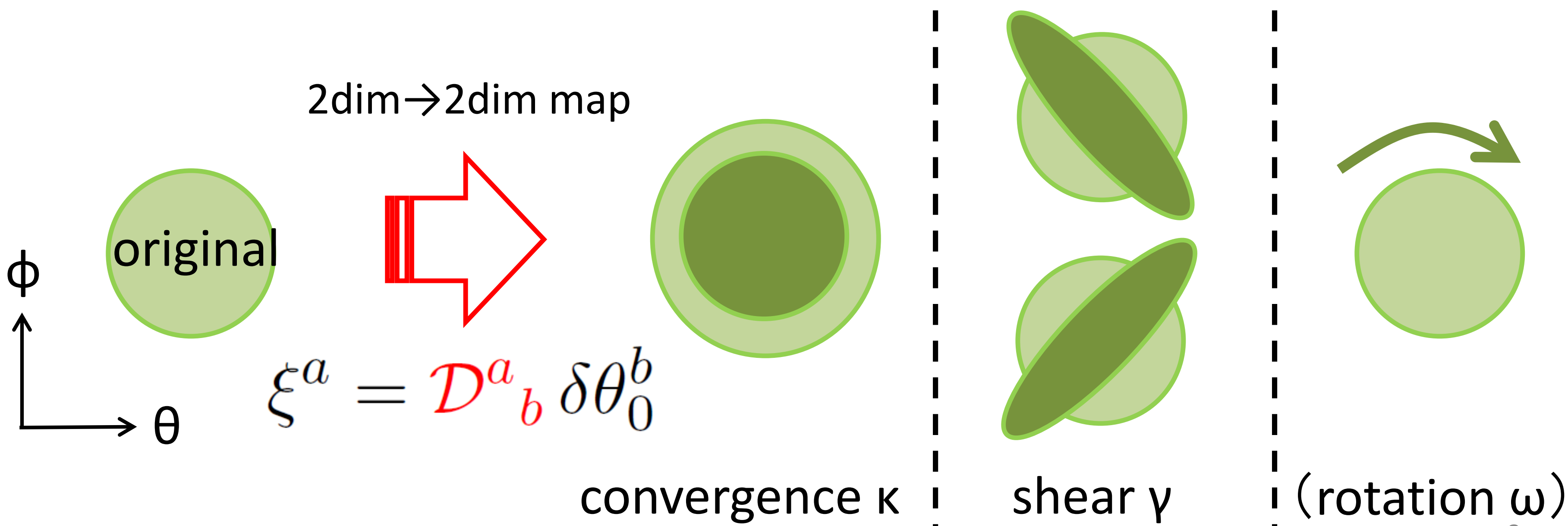
What we can measure in the shear measurement is the shape of galaxies modified by large scale structure.



Jacobi map

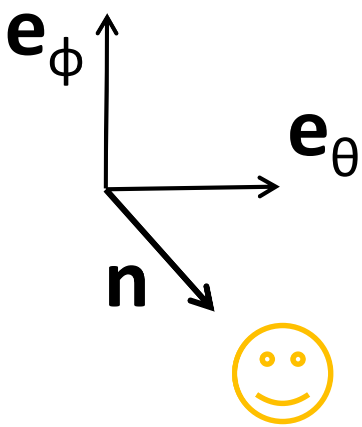
The shape of galaxies modified by lensing is characterized by the deformation of the two-dim spatial pattern:

$$\mathcal{D}^a{}_b = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2-\omega \\ -\gamma_2+\omega & 1-\kappa+\gamma_1 \end{pmatrix}$$



We should solve the Sachs equation for the Jacobi map:

$$\frac{\mathrm{d}^2}{\mathrm{d}\chi^2}\mathcal{D}^a{}_b = \mathcal{T}^a{}_c\,\mathcal{D}^c{}_b$$



symmetric optical tidal matrix

$$\mathcal{T}^a{}_b = -R_{\mu\rho\nu\sigma}\frac{\mathrm{d}x^\mu}{\mathrm{d}\chi}\frac{\mathrm{d}x^\nu}{\mathrm{d}\chi}e^{\rho a}e^\sigma_b$$

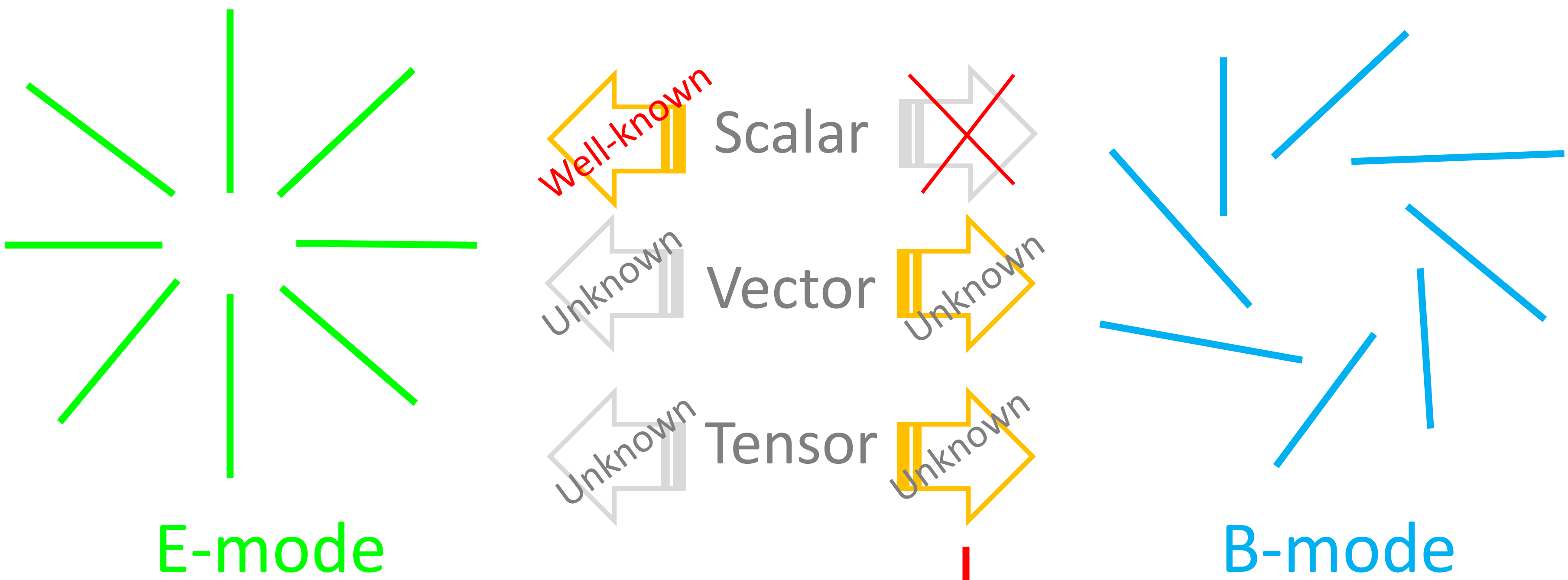
✓ Considering only scalar metric perturbations, the shear (trace-free part of Jacobi map) has the well-known form;

geometry grav. potential

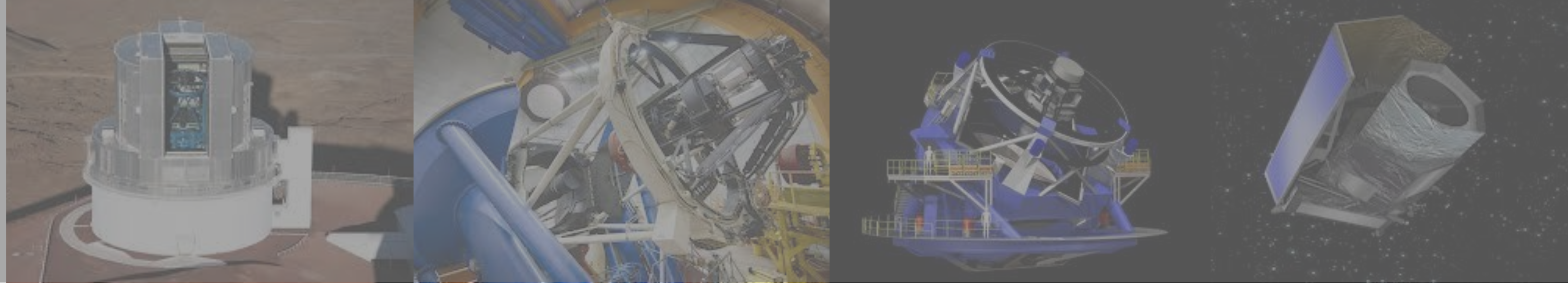
$$\gamma_{ab} = \frac{1}{\chi_\mathrm{S}}\mathcal{D}_{\langle ab\rangle} = \int_0^{\chi_\mathrm{S}}\mathrm{d}\chi\,\frac{\chi_\mathrm{S}-\chi}{\chi_\mathrm{S}\chi}\left(\Psi-\Phi\right)_{:\langle ab\rangle}$$

E-/B-mode decomposition

The shear field γ_{ab} can be decomposed into even-parity part (E-mode) and odd-parity part (B-mode).



The non-vanishing B-mode shear signal would be a direct evidence for non-scalar metric perturbations.



Imaging survey

e.g. HSC, DES, LSST, Euclid…

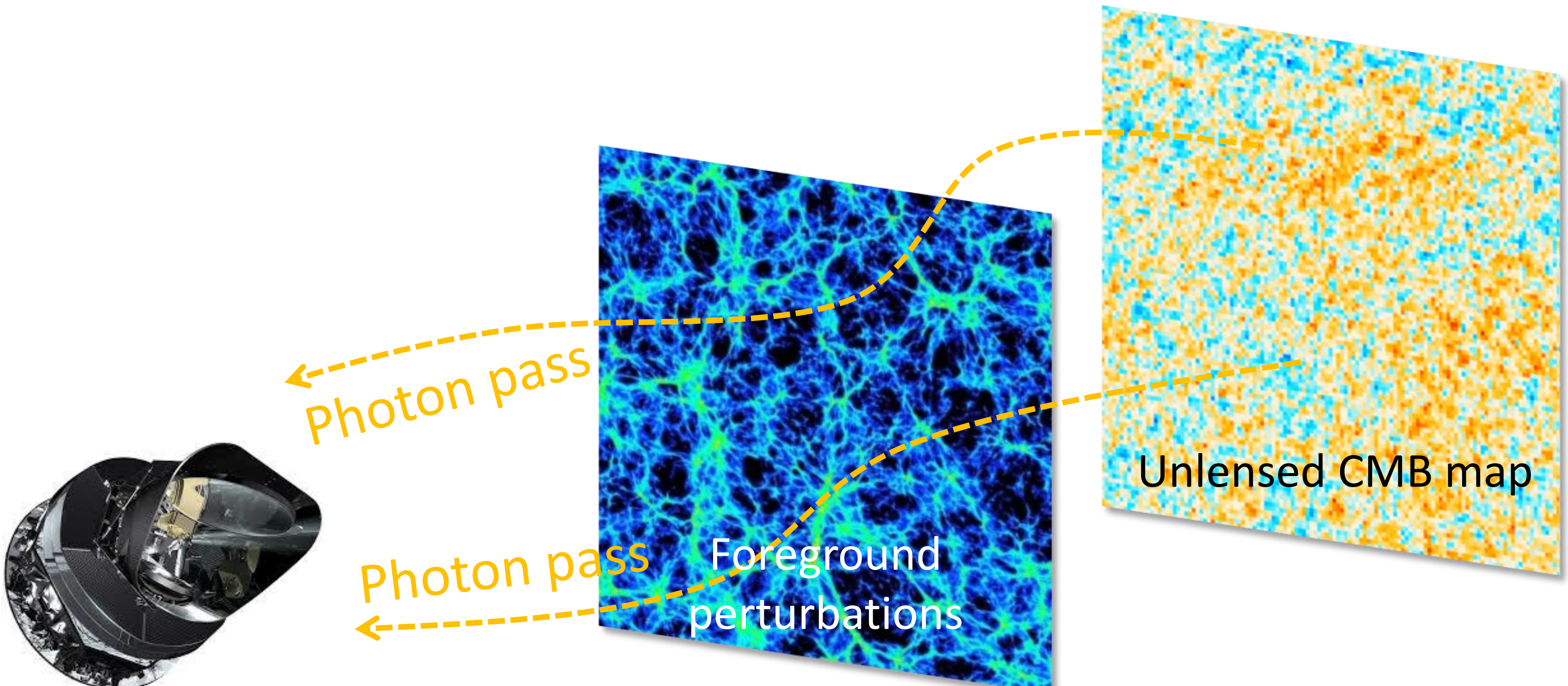
Weak lensing measurement

CMB lensing

e.g. ACT, SPT, Planck, PolarBear, ACTPol, SPTPol, COrE,…

CMB lensing

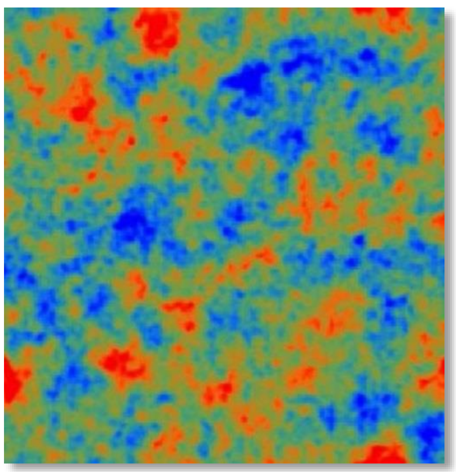
What we observe is a subtly distorted version of the primary CMB anisotropy.



Deflection field

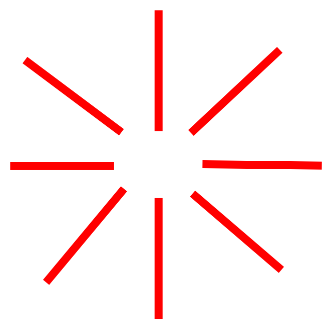
The distortion effect of lensing on the primary CMB is expressed by a remapping with the deflection angle "***Δ***".

$$\tilde{\Theta}(\hat{\boldsymbol{n}}) = \Theta(\hat{\boldsymbol{n}} + \boldsymbol{\Delta})$$



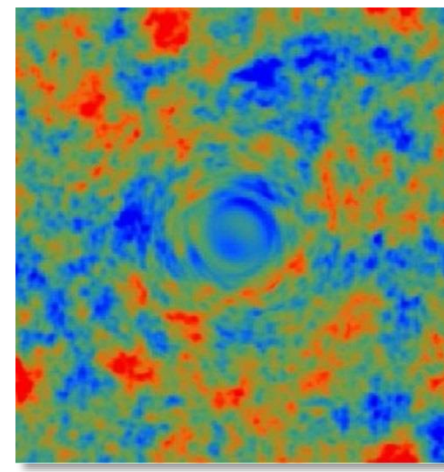
Unlensed : Θ

+



Deflection field:

$$\boldsymbol{\Delta} = \nabla\phi$$

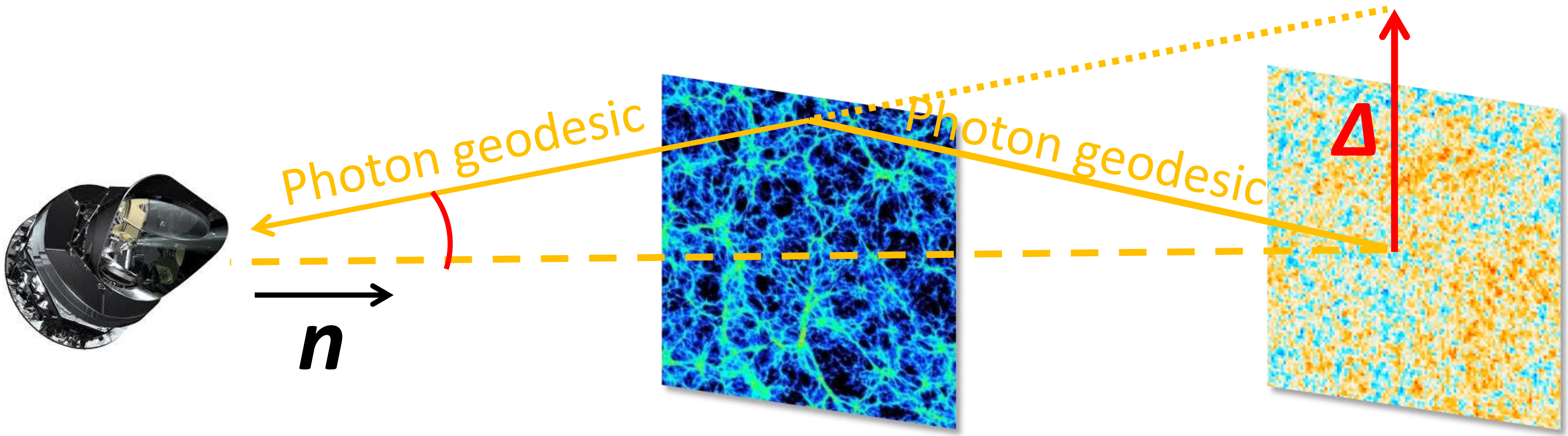


Lensed : $\tilde{\Theta}$

[figures : Hu+Okamoto(2002)]

Deflection field (contd.)

The apparent position modified by the deflection angle (that is what we really see) appears as a solution of the geodesic equation for the CMB photon.



✓ For scalar metric perturbations,

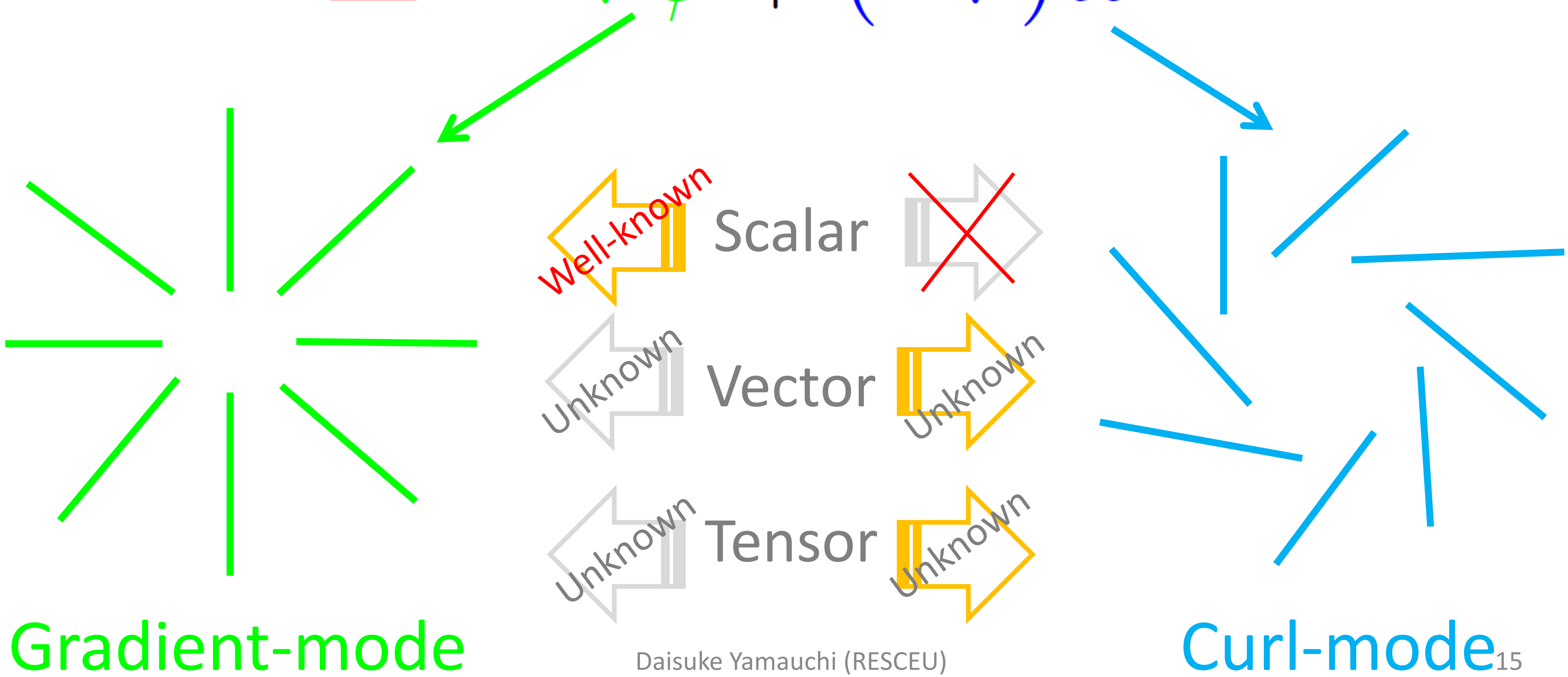
$$\Delta_a = \int_0^{\chi_S} d\chi \, \frac{\chi_S - \chi}{\chi_S \chi} \left(\Psi - \Phi \right)_{:a}$$

Only gradient-mode!

Gradient-/curl-mode decomposition

The two dimensional distortion vector $\boldsymbol{\Delta}$ is decomposed into gradient-mode: $\nabla\phi$ and curl-mode: $({}^*\nabla)\varpi$.

$$\boldsymbol{\Delta} = \nabla\phi + ({}^*\nabla)\varpi$$



Question

In usual treatment of lensing, the symmetric trace-free part of the gradient of the deflection Δ_a (from geodesic eq.) can be used as a proxy for the shear γ_{ab} (from Sachs eq.).

Is it always true?

$$\gamma_{ab} \stackrel{?}{=} \Delta_{\langle a:b \rangle}$$

NO!

SHEAR-DEFLECTION RELATION

Gauge-invariant metric perturbations

$$\delta g_{00} = -2A \,,$$

$$\delta g_{0i} = B_{|i} + B_i \,,$$

$$\delta g_{ij} = 2\mathcal{R}\delta_{ij} + 2H_{|ij} + 2H_{(i|j)} + h_{ij} \,,$$

⇩

Scalar metric pert.

$$\Phi \equiv A - \frac{1}{a}\left[a\left(\dot{H} - B\right)\right]^{\cdot}$$

$$\Psi \equiv \mathcal{R} - \frac{\dot{a}}{a}\left(\dot{H} - B\right)$$

Vector metric pert.

$$\sigma_{\mathrm{g},i} \equiv \dot{H}_i - B_i$$

Tensor metric pert.

$$h_{ij}$$

Deflection angle with gauge invariant variables

Solving the geodesic equation in the gauge-invariant manner, we obtain the deflection angle:

$$\Delta_a = \int_0^{\chi_S} \frac{\mathrm{d}\chi}{\chi} \left\{ \frac{\chi_S - \chi}{\chi_S} \Upsilon_{:a} - \Omega_a \right\} + \Omega_a \Big|_0$$

with

[see also Yoo (2009),...]

- ✓ spin-0 part $\Upsilon \equiv \Psi - \Phi + \sigma_{g,i}\hat{n}^i + \frac{1}{2}h_{ij}\hat{n}^i\hat{n}^j$
- ✓ spin-1 part $\Omega_a \equiv \sigma_{g,a} + h_{ia}\hat{n}^i$

Note : In the presence of the spin-1 part, that is vector σg and tensor h, there appears the curl-mode.

New !

Shear field with gauge invariant variables

Solving the Sachs equation for shear field, we have

spin-0 spin-1 spin-2

$$\gamma_{ab} = \int_0^{\chi_S} \frac{d\chi}{\chi} \left\{ \Upsilon_{:\langle ab\rangle} - \Omega_{\langle a:b\rangle} \right\} + \Omega_{\langle a:b\rangle}\Big|_0 + \frac{1}{2}\left(h_{\langle ab\rangle}\Big|_{\chi_S} - h_{\langle ab\rangle}\Big|_0 \right)$$

Since the gauge degrees of freedom are completely removed in the explicit expression for the tidal matrix, the resultant shear field are manifestly gauge-invariant.

New !

Gauge-inv. shear-deflection relation

Spin-2 contributions

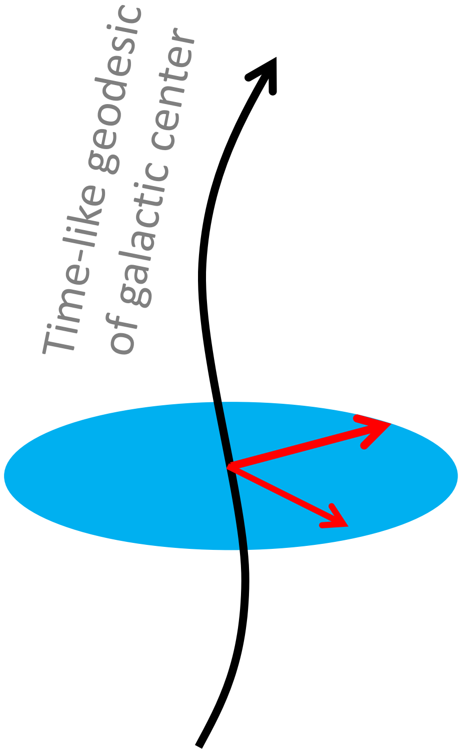
$$\gamma_{ab} = \Delta_{\langle a:b \rangle} + \frac{1}{2}\left(h_{\langle ab \rangle}\Big|_{\chi_S} - h_{\langle ab \rangle}\Big|_{0} \right)$$

✓ Metric shear [Dodelson+(2003)] / FNC term [Jeong+Schmidt(2012)] (based on the geodesic eq)

Cosmic shear measurement via galaxy survey are usually referenced to the coordinate in which galaxies are statistically isotropic. This is in general different from our reference coordinate (FLRW). The correction from the gravitational potential should appear.

In contrast to the previous studies, the metric shear/FNC term naturally arises in our case from the Sachs equation

Why does such correction appear in Sachs equation?



$$\delta g_{00}^{\mathrm{FNC}} = -R_{0\ell 0m} x^\ell x^m$$

$$\delta g_{0i}^{\mathrm{FNC}} = -\frac{2}{3} R_{0\ell im} x^\ell x^m$$

$$\delta g_{ij}^{\mathrm{FNC}} = -\frac{1}{3} R_{i\ell jm} x^\ell x^m$$

$$\Rightarrow \mathcal{T}_{ab}$$

Since the leading correction of the metric in the FNC is known to be described by the Riemann curvature, the FNC contribution is automatically included in the symmetric tidal matrix perturbed around FLRW.

LENSING POWER SPECTRA

In deriving lensing power spectrum,…

One complication is that while the weak lensing observables are defined on a spherical sky, the metric perturbations usually appear in the three-dim space.
Even decomposing the perturbations into the plane waves, they contribute to many multipole due to their angular structure…

E.g.) Bardeen potential

$$\Phi(\boldsymbol{x},\eta) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\Phi_{\boldsymbol{k}}(\eta) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \rightarrow \sum_{L=0}^{\infty} 4\pi(-i)^L j_L(k\chi) Y_L{}^0(\hat{\boldsymbol{n}})$$

The situation is more complicated for vector/tensor perturbations…

Total angular momentum wave

Combining the intrinsic angular structure with that of the plane-wave spatial dependence (TAM) substantially simplifies the derivation of the full-sky formula!

$$ {}_sG_\ell{}^m = \sum_{L=0}^{\infty} (-i)^L \sqrt{4\pi(2L+1)} \left({}_s\epsilon_L^{(\ell,m)}(k\chi) + i\,\mathrm{sgn}(s)\,{}_s\beta_L^{(\ell,m)}(k\chi) \right) {}_sY_L{}^m(\hat{\boldsymbol{n}}) $$

[originally developed by Hu+White (2001)]

It enables us to simultaneously treat the lensing by vector and tensor modes on an equal footing with those generated by scalar modes.

TAM expansion

- The m=0, +-1, +-2 modes corresponds to the scalar, vector, and tensor metric perturbations.
- We can isolate the total angular dependence by each perturbation.

✓ Spin-0 component

$$\Upsilon = \sum_{L=0}^{\infty} \sum_{m=-2}^{2} \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} (-i)^L \sqrt{4\pi(2L+1)}\, \Upsilon_{\boldsymbol{k}}^{(m)}\, {}_0 j_L^{(|m|,m)}(k\chi)\, {}_0 Y_L{}^m(\hat{\boldsymbol{n}})$$

✓ Spin-1 component

$$\Omega_a e_{\pm}^a = \sum_{L=0}^{\infty} \sum_{m=-2}^{2} \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} (-i)^L \sqrt{4\pi(2L+1)}\, {}_{\pm 1}\Omega_{\boldsymbol{k}}^{(m)}\, {}_{\pm 1} j_L^{(|m|,m)}(k\chi)\, {}_{\pm 1} Y_L{}^m(\hat{\boldsymbol{n}})$$

✓ Spin-2 component

$$h_{ab} e_{\pm}^a e_{\pm}^b = \sum_{L=0}^{\infty} \sum_{m=-2}^{2} \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} (-i)^L \sqrt{4\pi(2L+1)}\, {}_{\pm 2} h_{\boldsymbol{k}}^{(m)}\, {}_{\pm 2} j_L^{(|m|,m)}(k\chi)\, {}_{\pm 2} Y_L{}^m(\hat{\boldsymbol{n}})$$

New !

A complete set of full-sky formula for scalar/vector/tensor metric pert.

The m=0, +-1, +-2 modes corresponds to the scalar, vector, tensor metric perturbations.

$$C_\ell^{\mathrm{XX}'} = \frac{2}{\pi}\sum_{m=-2}^{2}\int_0^\infty \mathrm{d}k\,k^2\int_0^\infty \mathrm{d}\chi\int_0^\infty \mathrm{d}\chi'\mathcal{S}_{\mathrm{X},\ell}^{(m)}(k,\chi)\mathcal{S}_{\mathrm{X}',\ell}^{(m)}(k,\chi')\mathcal{P}_{|m|}(k,\chi,\chi')$$

Auto-power spectrum for m-mode

- Transfer functions for X and X'

New !

Radial transfer functions for E-/B-mode cosmic shear

➢ E-mode shear

$N(\chi_S)$: source redshift distribution

$$\mathcal{S}^{(0)}_{\mathrm{E},\ell} = \sqrt{\frac{(\ell-2)!}{(\ell+2)!}}\frac{1}{k\chi}\int_\chi^\infty \mathrm{d}\chi_\mathrm{S}\frac{\chi_\mathrm{S}-\chi}{\chi_\mathrm{S}}\frac{N(\chi_\mathrm{S})}{N_\mathrm{g}}\,{}_0\epsilon^{(0,0)}_\ell(k\chi)$$

$$\mathcal{S}^{(\pm1)}_{\mathrm{E},\ell} = \frac{1}{2}\sqrt{\frac{(\ell-2)!}{(\ell+2)!}}\frac{1}{k\chi}\int_\chi^\infty \mathrm{d}\chi_\mathrm{S}\frac{N(\chi_\mathrm{S})}{N_\mathrm{g}}\left[\frac{\chi_\mathrm{S}-\chi}{\chi_\mathrm{S}}\,{}_0\epsilon^{(1,\pm1)}_\ell(k\chi) - \sqrt{2\frac{(\ell-1)!}{(\ell+1)!}}\,{}_1\epsilon^{(1,\pm1)}_\ell(k\chi)\right]$$

$$\mathcal{S}^{(\pm2)}_{\mathrm{E},\ell} = \frac{1}{4}\sqrt{\frac{(\ell-2)!}{(\ell+2)!}}\frac{1}{k\chi}\int_\chi^\infty \mathrm{d}\chi_\mathrm{S}\frac{N(\chi_\mathrm{S})}{N_\mathrm{g}}\left[\frac{1}{\sqrt{3}}\frac{\chi_\mathrm{S}-\chi}{\chi_\mathrm{S}}\,{}_0\epsilon^{(2,\pm2)}_\ell(k\chi) - \sqrt{2\frac{(\ell-1)!}{(\ell+1)!}}\,{}_1\epsilon^{(2,\pm2)}_\ell(k\chi)\right]$$
$$+\frac{1}{10\sqrt{2}}\delta_{\ell,2}\delta_\mathrm{D}(k\chi) + \frac{1}{2\sqrt{2}}\frac{N(\chi)}{kN_\mathrm{g}}\,{}_2\epsilon^{(2,\pm2)}_\ell(k\chi)$$

Metric shear/FNC term (from spin-2)

➢ B-mode shear

$$\mathcal{S}^{(0)}_{\mathrm{B},\ell} = 0$$

$$\mathcal{S}^{(\pm1)}_{\mathrm{B},\ell} = \cdots$$

$$\mathcal{S}^{(\pm2)}_{\mathrm{B},\ell} = \cdots$$

We explicitly show that B-mode is not generated by the m=0 mode (scalar metric perturbations), as is expected.

New !

Radial transfer functions for gradient-/curl-mode CMB-lensing

- gradient mode

$$\mathcal{S}^{(0)}_{\phi,\ell} = 2\frac{\chi_{\rm S}-\chi}{\chi_{\rm S}}\frac{1}{k\chi}\,{}_0\epsilon^{(0,0)}_\ell(k\chi)\,,$$

$$\mathcal{S}^{(\pm1)}_{\phi,\ell} = \frac{1}{k\chi}\left[\frac{\chi_{\rm S}-\chi}{\chi_{\rm S}}\,{}_0\epsilon^{(1,\pm1)}_\ell(k\chi) - \sqrt{2\frac{(\ell-1)!}{(\ell+1)!}}\,{}_1\epsilon^{(1,\pm1)}_\ell(k\chi)\right]$$

$$\mathcal{S}^{(\pm2)}_{\phi,\ell} = \frac{1}{2k\chi}\left[\frac{\chi_{\rm S}-\chi}{\sqrt{3}\chi_{\rm S}}\,{}_0\epsilon^{(2,\pm2)}_\ell(k\chi) - \sqrt{2\frac{(\ell-1)!}{(\ell+1)!}}\,{}_1\epsilon^{(2,\pm2)}_\ell(k\chi)\right] + \frac{1}{10\sqrt{3}}\delta_{\ell,2}\delta_{\rm D}(k\chi)$$

- Curl mode

$$\mathcal{S}^{(0)}_{\varpi,\ell} = 0$$

$\mathcal{S}^{(\pm1)}_{\varpi,\ell} =$

$\mathcal{S}^{(\pm2)}_{\varpi,\ell} =$

Curl-mode is not generated by the m=0 mode (scalar metric perturbations), as is expected.

APPLICATION

Primordial gravitational wave

As an application for the *curl-/B-mode*, let us consider the primordial gravitational wave as the representative passive sources for tensor perturbations.

- the tensor metric perturbations ⇔ m=+-2 modes

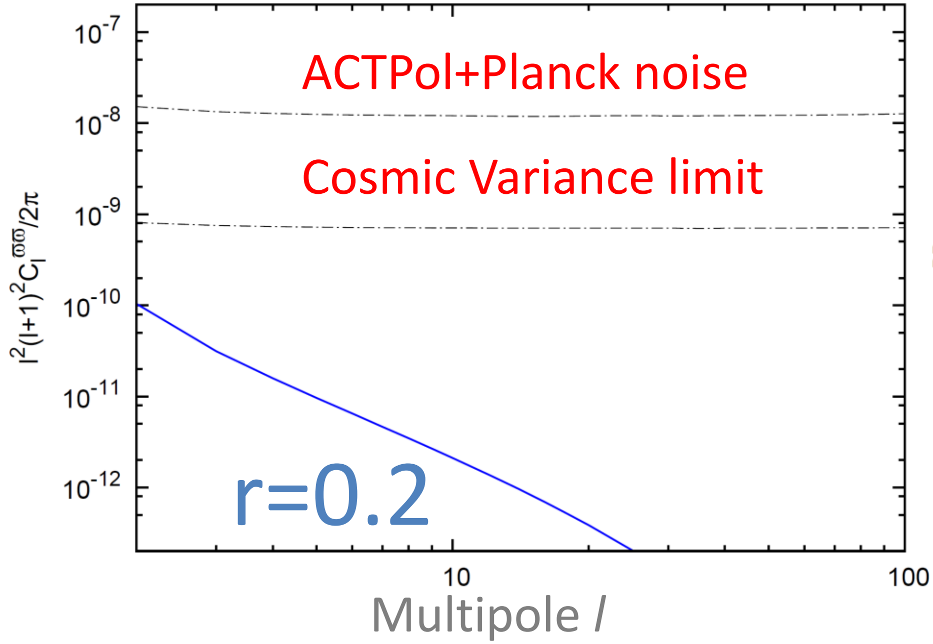
$$C_\ell^{\varpi\varpi,\mathrm{PGW(T)}} = 4\pi\int_0^\infty \frac{\mathrm{d}k}{k}\,\Delta_h^2(k)\left[\frac{1}{2}\frac{(\ell-1)!}{(\ell+1)!}\sqrt{\frac{(\ell+2)!}{(\ell-2)!}}\int_0^{\chi_\mathrm{S}}\mathrm{d}\chi\frac{3j_1(k(\eta_0-\chi))}{k(\eta_0-\chi)}\frac{j_\ell(k\chi)}{k\chi^2}\right]^2$$

$$C_\ell^{\mathrm{BB,PGW(T)}} = 4\pi\int_0^\infty \frac{\mathrm{d}k}{k}\,\Delta_h^2(k)\left[\frac{1}{4}\int_0^\infty\mathrm{d}\chi\frac{3j_1(k(\eta_0-\chi))}{k(\eta_0-\chi)}\right.$$
$$\left.\times\left\{\frac{(\ell+2)!(\ell-1)!}{(\ell-2)!(\ell+1)!}\int_\chi^\infty\mathrm{d}\chi_\mathrm{S}\frac{N(\chi_\mathrm{S})}{N_\mathrm{g}}\frac{j_\ell(k\chi)}{k\chi^2}-\frac{N(\chi)}{N_\mathrm{g}}\left(j_\ell'(k\chi)+2\frac{j_\ell(k\chi)}{k\chi}\right)\right\}\right]^2$$

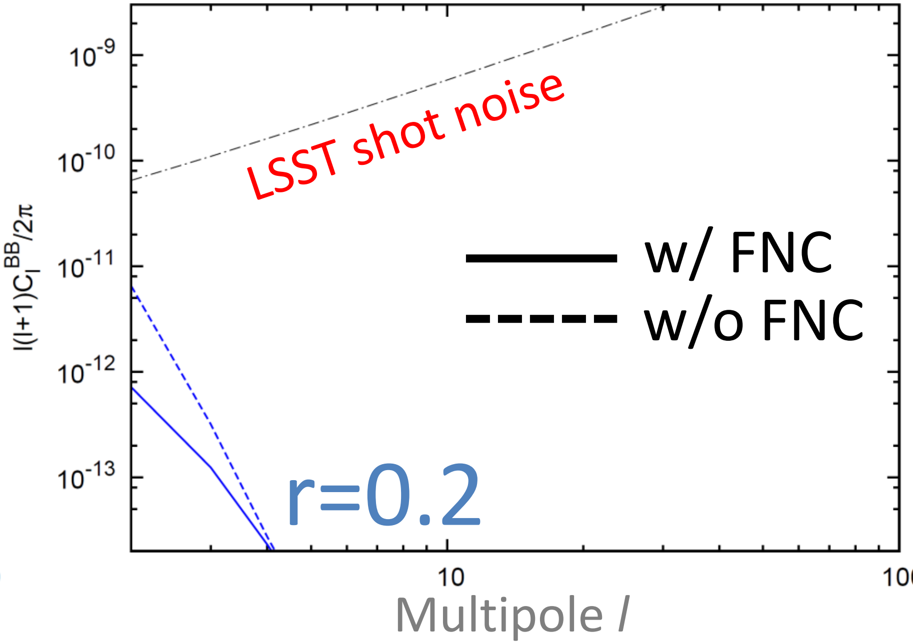
The contribution from the PGW (m=+-2) are shown to coincide with those derived by other authors [Dodelson+(2003), Schmidt+Jeong(2012)].

Primordial gravitational wave

✓ **Curl-mode of CMB lensing**



✓ **B-mode shear**



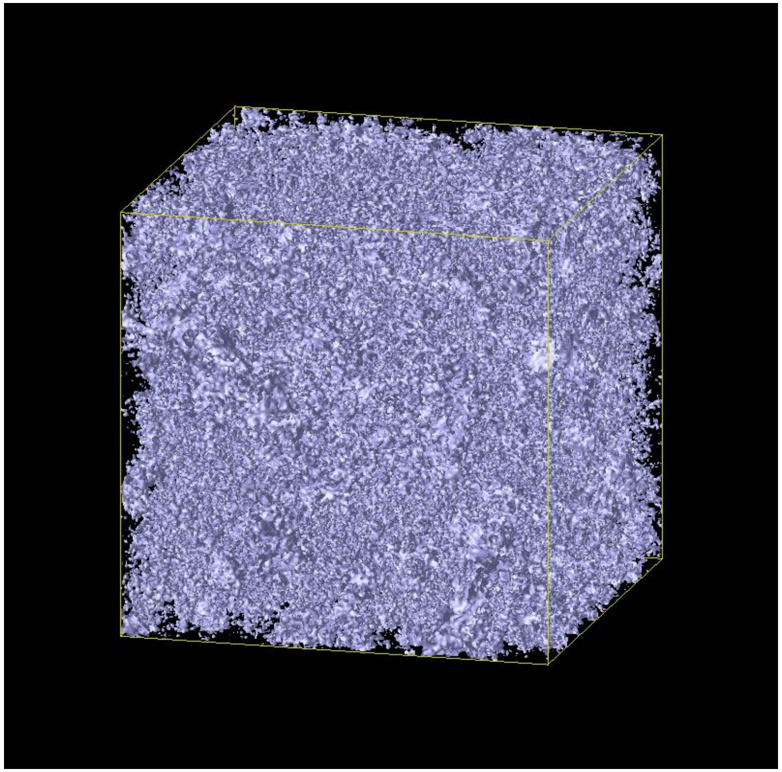
Comparison between the statistical errors and the predictions immediately follows that it is challenging to detect PGW via the weak lensing measurement.

(Full-sky curl-mode estimator [Namikawa+**DY**+Taruya, 1110.1718])

Cosmic strings

We consider the cosmic strings as intriguing examples of the active source for vector and tensor perturbations.

(In this talk, I use the simple model for the string network to calculate analytically.)



- Line-like topological defects
- generally form during phase transition in the very early universe. [Jeannerot+(2003)]
- could be a probe for the early phases of the universe before the CMB epoch

[Hiramatsu+Sendouda+Takahashi+**DY**+Yoo, 1307.0308]

Cosmic strings

We consider the cosmic strings as intriguing examples of the active source for vector and tensor perturbations.

(In this talk, I use the simple model for the string network to calculate analytically.)

- the vector/tensor perturbations ⇔ m=+-1, +-2 modes

$$C_\ell^{\varpi\varpi,\mathrm{CS(V)}} = 4\pi\int_0^\infty \frac{\mathrm{d}k}{k}\left[\sqrt{\frac{(\ell-1)!}{(\ell+1)!}}\int_0^{\chi_\mathrm{S}}\frac{\mathrm{d}\chi}{\chi}\Delta_1(k,\chi)\,j_\ell(k\chi)\right]^2,$$

$$C_\ell^{\varpi\varpi,\mathrm{CS(T)}} = 4\pi\int_0^\infty \frac{\mathrm{d}k}{k}\left[\frac{1}{2}\frac{(\ell-1)!}{(\ell+1)!}\sqrt{\frac{(\ell+2)!}{(\ell-2)!}}\int_0^{\chi_\mathrm{S}}\mathrm{d}\chi\Delta_2(k,\chi)\frac{j_\ell(k\chi)}{k\chi^2}\right]^2$$

$$C_\ell^{\mathrm{BB,CS(V)}} = 4\pi\int_0^\infty \frac{\mathrm{d}k}{k}\left[\frac{1}{2}\sqrt{\frac{(\ell+2)!(\ell-1)!}{(\ell-2)!(\ell+1)!}}\int_0^\infty\frac{\mathrm{d}\chi}{\chi}\int_\chi^\infty\mathrm{d}\chi_\mathrm{S}\frac{N(\chi_\mathrm{S})}{N_\mathrm{g}}\Delta_1(k,\chi)j_\ell(k\chi)\right]^2$$

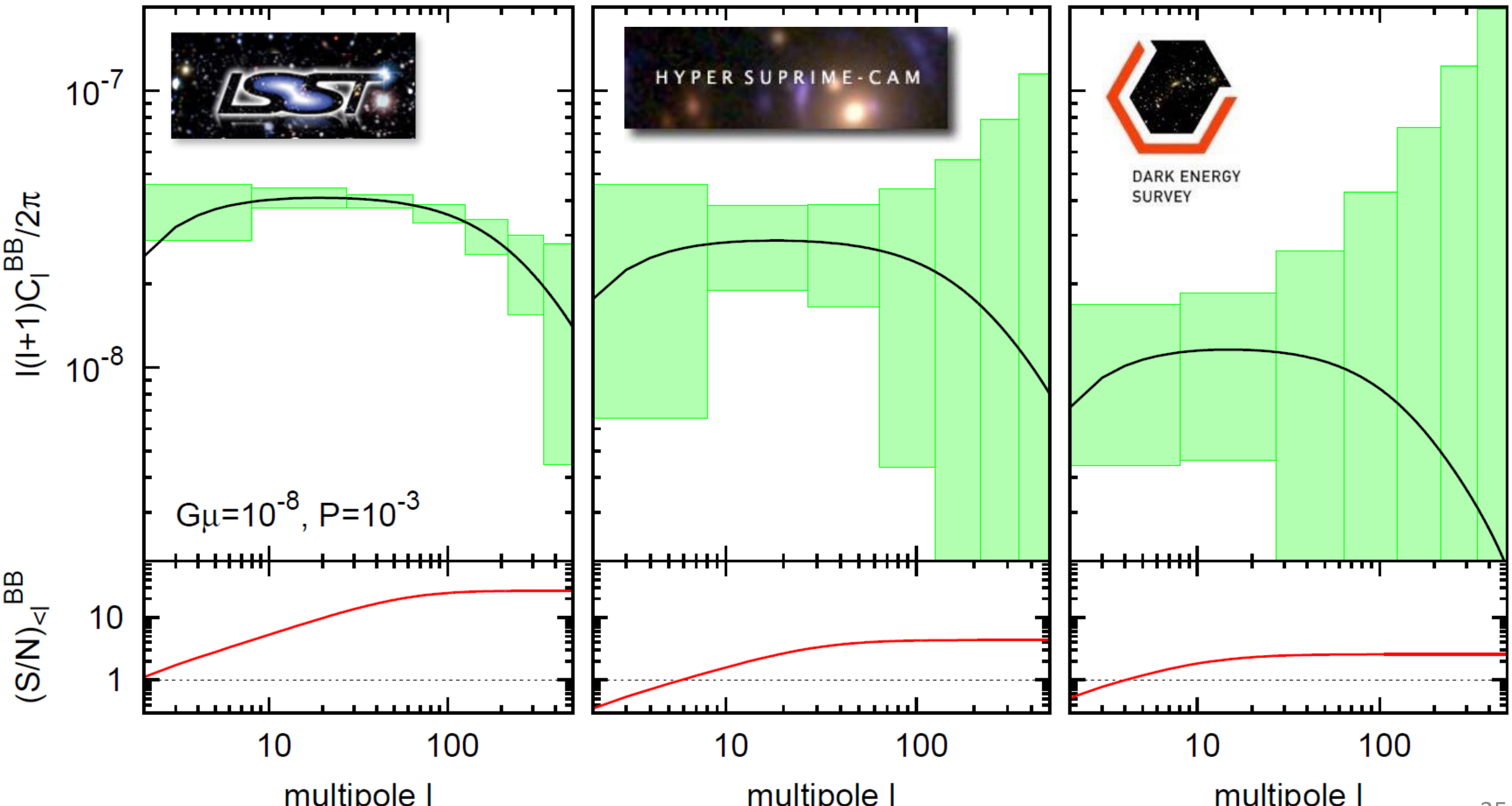
$$C_\ell^{\mathrm{BB,CS(T)}} = 4\pi\int_0^\infty \frac{\mathrm{d}k}{k}\left[\frac{1}{4}\int_0^\infty\mathrm{d}\chi\Delta_2(k,\chi)\right.$$
$$\left.\times\left\{\frac{(\ell+2)!(\ell-1)!}{(\ell-2)!(\ell+1)!}\int_\chi^\infty\mathrm{d}\chi_\mathrm{S}\frac{N(\chi_\mathrm{S})}{N_\mathrm{g}}\frac{j_\ell(k\chi)}{k\chi^2}-\frac{N(\chi)}{N_\mathrm{g}}\left(j'_\ell(k\chi)+2\frac{j_\ell(k\chi)}{k\chi}\right)\right\}\right]^2$$

New !

B-mode shear from cosmic strings

[**DY**+Namikawa+Taruya, 1205.2139, 1305.3348]

Fiducial string parameters : String tension **Gμ =10^{-8},** reconnection prob. **P= 10^{-3}**

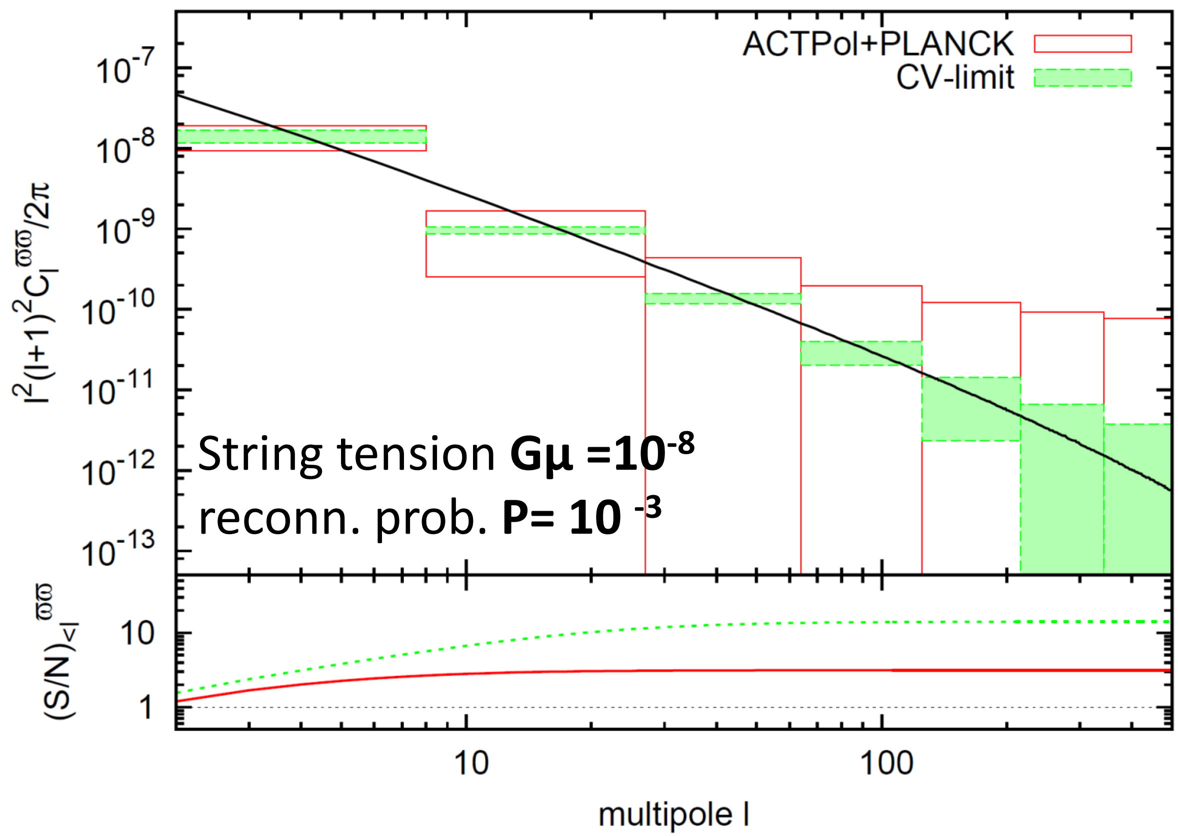


New !

(Full-sky curl-mode estimator [Namikawa+**DY**+Taruya 1110.1718])

Curl-deflection from cosmic strings

[**DY**+Namikawa+Taruya, 1205.2139, 1305.3348]

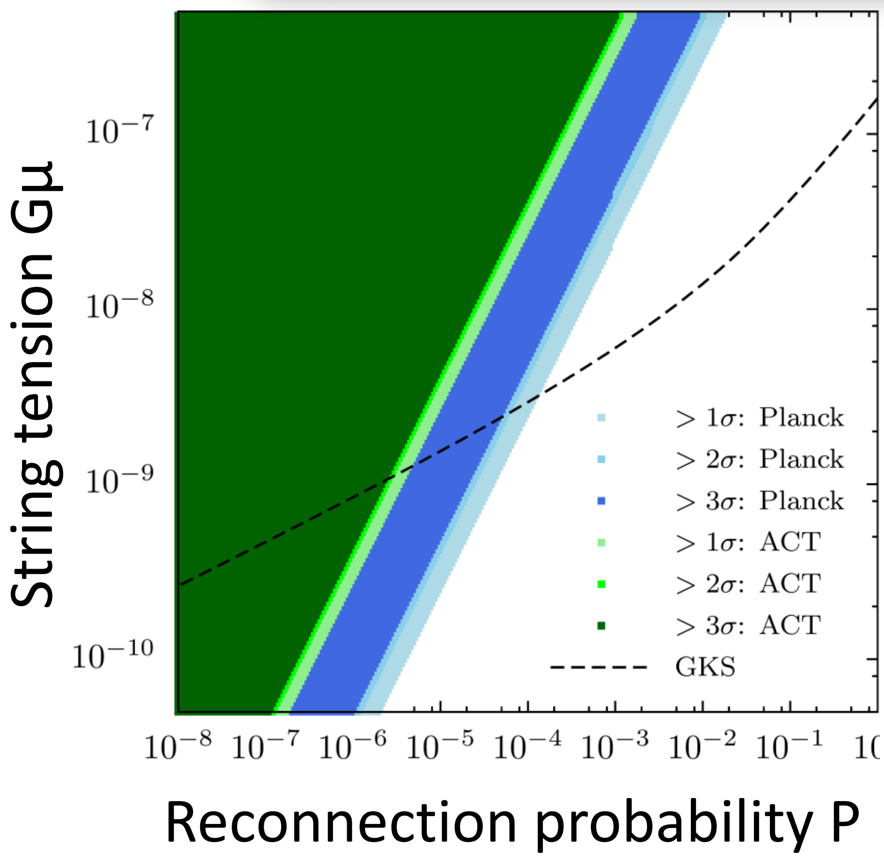


The curl-mode measurement would provide not only a direct probe of cosmic strings, but also a diagnosis helpful to check the systematics in the derived constraints from the CMB TT.

New !

Constraint on string parameters from curl mode for ACT and Planck

$$G\mu P^{-1} \leq 3.4 \times 10^{-5}$$ (95%CL, Planck)



Lensing curl mode is more sensitive to small values of reconnection probability P compared to the small scale temperature power spectrum.

[Namikawa+**DY**+Taruya, 1308.6068]

Summary

1. Gauge-invariant deflection-shear relation

$$\gamma_{ab} = \Delta_{\langle a:b \rangle} + \frac{1}{2}\left(h_{\langle ab \rangle}\Big|_{\chi_{\rm S}} - h_{\langle ab \rangle}\Big|_{0} \right)$$

In contrast previous studies, the metric shear/FNC term naturally arises in our case from Sachs eq.

2. Lensing power spectra with TAM

Total angular momentum method substantially simplifies the derivation of the full-sky formula.

Thank you !