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バブル生成を伴うインフレーション模型における電磁場の超曲率モード は、ない

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JCAP03(2014)031, 1402.2784, **DY**, T. Fujita(KIPMU), S.Mukohyama(KIPMU)

原始磁場

- ✓ $B = O(10^{-14}\text{-}10^{-20})$G
- ✓ $L = O(1)$Mpc
- ✓ Void region

[Neronov+Vovk (2010),...]

⇩

原始磁場の生成機構があるはず（？）

インフレーションの文脈で理解することができるか？

磁場の発展 (空間曲率無のとき)

$$\left(a^2 B_i\right)'' + k^2 a^2 B_i = 0$$

⇓

$$B_i \propto \frac{1}{a^2} \exp\left[\pm ik\eta\right]$$

磁場はどんな波数モードも$1/a^2$で急速に減衰する

磁場の発展 (空間曲率有のとき)

✓ 平坦FLRWのとき

$$(a^2 B_i)'' + k^2 a^2 B_i = 0$$

$$B_i \propto \frac{1}{a^2} \exp\left[\pm i k \eta \right]$$

✓ 空間曲率有のとき

$$(a^2 B_i)'' + k^2 a^2 B_i + a^2 R_i{}^j B_j = 0$$

定曲率空間

$$(a^2 B_i)'' + \left(k^2 + \frac{2K}{r_{\rm curv}^2} \right) a^2 B_i = 0$$

$$B_i \propto \frac{1}{a^2} \exp\left[\pm i \sqrt{k^2 + \frac{2K}{r_{\rm curv}^2}}\, \eta \right]$$

Superadiabatic amplification

- 開いた宇宙(K=-1)の場合

$$B_i \propto \frac{1}{a^2} \exp\left[\sqrt{\frac{2}{r_{\rm curv}^2} - k^2}\, \eta \right]$$

もしも、曲率半径より大きい波数モード ($k_{SC}^2 < 2/r_{curv}^2$)が存在したとすると・・・

磁場が一部のモードで増幅！

⇨ 原始磁場の問題を解決する糸口になる（？）

[Barrow+Tsagas+Yamamoto(2012), Shtanov+Sahni(2013),...]

疑問

そもそも

ゲージ場の超曲率モードは
存在するのか？

結論

自発的に対称性を破っても破らなくても

ない

（少なくともオープンインフレーションの文脈において）

[**DY**+Fujita+Mukohyama (2013)]

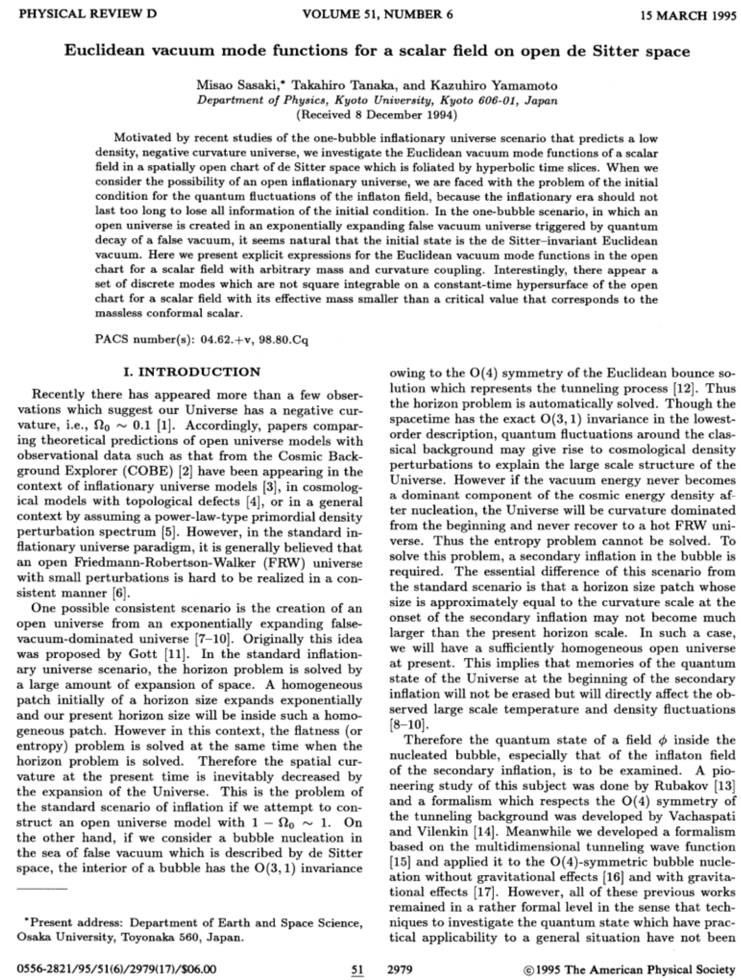
しかし、古の時代の結果によると…

✓ Sasaki+Tanaka+Yamamoto (1994)

「質量の軽いスカラー場は
1つだけ超曲率モードを持つ」

と書いてある

ありそう･･･



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Euclidean vacuum mode functions for a scalar field on open de Sitter space

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Motivated by recent studies of the one-bubble inflationary universe scenario that predicts a low density, negative curvature universe, we investigate the Euclidean vacuum mode functions of a scalar field in a spatially open chart of de Sitter space which is foliated by hyperbolic time slices. When we consider the possibility of an open inflationary universe, we are faced with the problem of the initial condition for the quantum fluctuations of the inflaton field, because the inflationary era should not last too long to lose all information of the initial condition. In the one-bubble scenario, in which an open universe is created in an exponentially expanding false vacuum universe triggered by quantum decay of a false vacuum, it seems natural that the initial state is the de Sitter–invariant Euclidean vacuum. Here we present explicit expressions for the Euclidean vacuum mode functions in the open chart for a scalar field with arbitrary mass and curvature coupling. Interestingly, there appear a set of discrete modes which are not square integrable on a constant-time hypersurface of the open chart for a scalar field with its effective mass smaller than a critical value that corresponds to the massless conformal scalar.

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I. INTRODUCTION

Recently there has appeared more than a few observations which suggest our Universe has a negative curvature, i.e., $\Omega_0 \sim 0.1$ [1]. Accordingly, papers comparing theoretical predictions of open universe models with observational data such as that from the Cosmic Background Explorer (COBE) [2] have been appearing in the context of inflationary universe models [3], in cosmological models with topological defects [4], or in a general context by assuming a power-law-type primordial density perturbation spectrum [5]. However, in the standard inflationary universe paradigm, it is generally believed that an open Friedmann-Robertson-Walker (FRW) universe with small perturbations is hard to be realized in a consistent manner [6].

One possible consistent scenario is the creation of an open universe from an exponentially expanding false-vacuum-dominated universe [7–10]. Originally this idea was proposed by Gott [11]. In the standard inflationary universe scenario, the horizon problem is solved by a large amount of expansion of space. A homogeneous patch initially of a horizon size expands exponentially and our present horizon size will be inside such a homogeneous patch. However in this context, the flatness (or entropy) problem is solved at the same time when the horizon problem is solved. Therefore the spatial curvature at the present time is inevitably decreased by the expansion of the Universe. This is the problem of the standard scenario of inflation if we attempt to construct an open universe model with $1 - \Omega_0 \sim 1$. On the other hand, if we consider a bubble nucleation in the sea of false vacuum which is described by de Sitter space, the interior of a bubble has the O(3, 1) invariance owing to the O(4) symmetry of the Euclidean bounce solution which represents the tunneling process [12]. Thus the horizon problem is automatically solved. Though the spacetime has the exact O(3, 1) invariance in the lowest-order description, quantum fluctuations around the classical background may give rise to cosmological density perturbations to explain the large scale structure of the Universe. However if the vacuum energy never becomes a dominant component of the cosmic energy density after nucleation, the Universe will be curvature dominated from the beginning and never recover to a hot FRW universe. Thus the entropy problem cannot be solved. To solve this problem, a secondary inflation in the bubble is required. The essential difference of this scenario from the standard scenario is that a horizon size patch whose size is approximately equal to the curvature scale at the onset of the secondary inflation may not become much larger than the present horizon scale. In such a case, we will have a sufficiently homogeneous open universe at present. This implies that memories of the quantum state of the Universe at the beginning of the secondary inflation will not be erased but will directly affect the observed large scale temperature and density fluctuations [8–10].

Therefore the quantum state of a field ϕ inside the nucleated bubble, especially that of the inflaton field of the secondary inflation, is to be examined. A pioneering study of this subject was done by Rubakov [13] and a formalism which respects the O(4) symmetry of the tunneling background was developed by Vachaspati and Vilenkin [14]. Meanwhile we developed a formalism based on the multidimensional tunneling wave function [15] and applied it to the O(4)-symmetric bubble nucleation without gravitational effects [16] and with gravitational effects [17]. However, all of these previous works remained in a rather formal level in the sense that techniques to investigate the quantum state which have practical applicability to a general situation have not been

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超曲率モードチェック項目

- [x] 1. 2次の作用を物理的自由度のみで書き下したか
- [x] 2. Klein-Gordonノルムは適切に定義したか
- [x] 3. モードは規格化可能になっているか

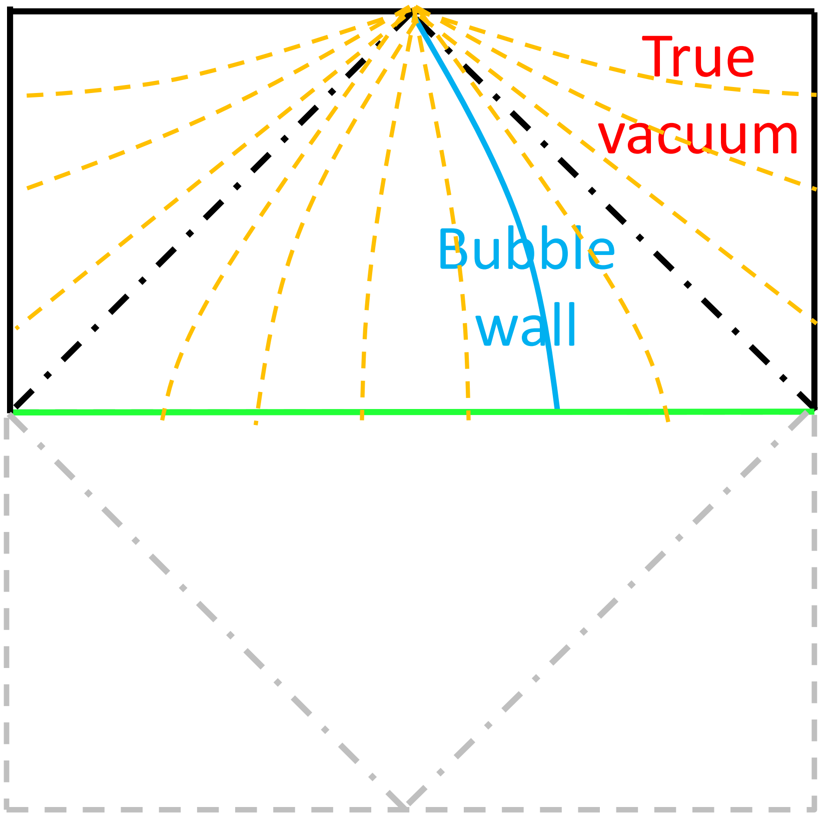
⇨ ゲージ場に超曲率モードは存在

セットアップ

- 量子化の議論をするためには背景時空を決める必要がある

✓ バブル生成を伴うインフレーション (open inflation)

-- バブル内部はK=-1(open) FLRW



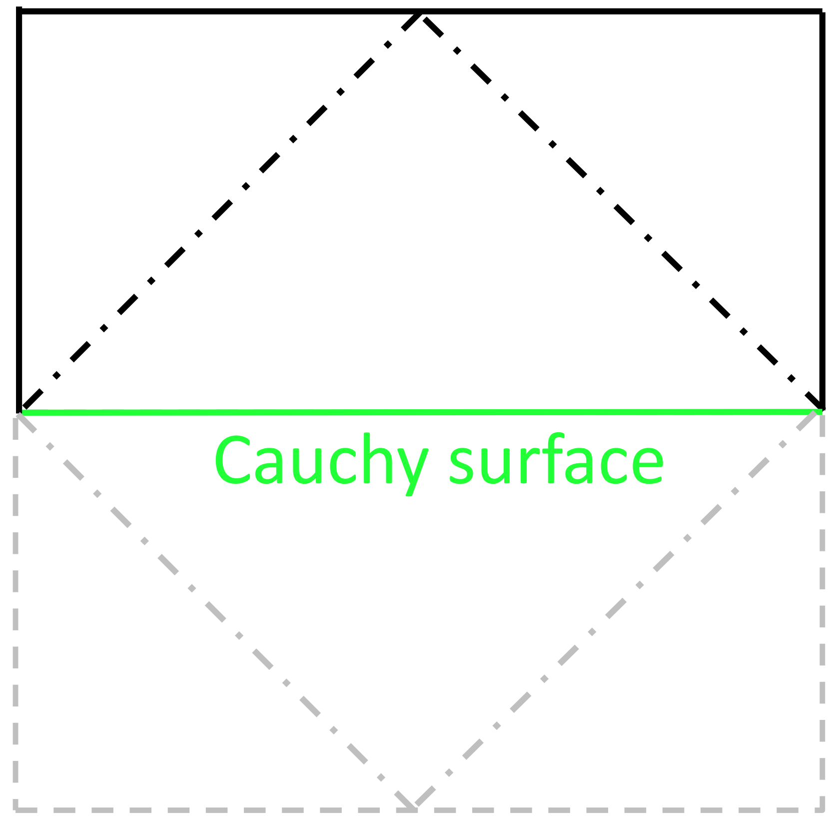
困難1：対称性が低い

量子化は「Cauchy面」上で行わなければいけない！

✓コーシー面を含む領域の計量

$$ds^2 = a^2(\eta)\left[d\eta^2 - dr^2 + \cosh^2 r d\Omega\right]$$

- 対称性が低い！
- η：空間座標, r:時間座標



困難2：モード混合の可能性

$$\mathrm{d}s^2 = a^2(\eta)\left[\mathrm{d}\eta^2 - \mathrm{d}r^2 + \cosh^2 r \mathrm{d}\Omega\right]$$

⇒ スカラー/ベクトル/テンソルモードが混じる可能性

⇒ スカラーモードの超曲率モードがベクトルモードにも伝播しうる!

全てのモードを同時に解く必要がある
（通常の宇宙論的摂動の手法は使えない）

困難3：補助場の存在

✓ Massive U(1) gauge theory (Proca theory)

$$S = -\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + m_A^2 g^{\mu\nu}A_\mu A_\nu\right]$$

➢ A_rが補助場として振舞う → 拘束条件を解く必要性

$$A_r^{\ell m} = \hat{\mathcal{O}}^{-1}\frac{\ell(\ell+1)}{\cosh^2 r}\partial_r V^{(\mathrm{e})\ell m} - \hat{\mathcal{K}}^{-1}\partial_\eta\partial_r A_\eta^{\ell m}$$

ここで

$$\begin{cases} \mathcal{O} = -\partial_\eta^2 + m_A^2 a^2 + \dfrac{\ell(\ell+1)}{\cosh^2 r} \\ \mathcal{K} = -\partial_\eta^2 + m_A^2 a^2 \end{cases}$$

チェック項目1：作用を縮約

r:時間的
η：空間的

- Even-parity scalar mode

$$S_{\rm s}^{(\rm e)} = \frac{1}{2}\sum_{\ell m}\int {\rm d}r{\rm d}\eta \Big\{ \cosh^2 r \left(\partial_r A_\eta^{\ell m}\right)\left[1+\partial_\eta \hat{\mathcal{K}}^{-1}\partial_\eta\right]\left(\partial_r A_\eta^{\ell m}\right) - \ell(\ell+1)A_\eta^{\ell m}\left[1+\partial_\eta \hat{\mathcal{K}}^{-1}\partial_\eta\right]A_\eta^{\ell m} - m_A^2 a^2 \cosh^2 r \left(A_\eta^{\ell m}\right)^2 \Big\}$$

- Even-parity vector mode

$$S_{\rm v}^{(\rm e)} = \frac{1}{2}\sum_{\ell m}\ell(\ell+1)\int {\rm d}r{\rm d}\eta \Big\{ \left(\partial_r V^{(\rm e)\ell m}\right)\left[1-\hat{\partial}^{-1}\frac{\ell(\ell+1)}{\cosh^2 r}\right]\left(\partial_r V^{(\rm e)\ell m}\right) - V^{(\rm e)\ell m}\hat{\mathcal{K}}V^{(\rm e)\ell m} \Big\}$$

- Odd-parity mode

$$S^{(\rm o)} = \frac{1}{2}\sum_{\ell m}\ell(\ell+1)\int {\rm d}r{\rm d}\eta \Big\{ \left(\partial_r A^{(\rm o)\ell m}\right)^2 - \left(\partial_\eta A^{(\rm o)\ell m}\right)^2 - \left(\frac{\ell(\ell+1)}{\cosh^2 r} + a^2 m_A^2\right)\left(A^{(\rm o)\ell m}\right)^2 \Big\}$$

スカラー/ベクトルモードが完全に分離！
[困難1,2を回避]

チェック項目2：KGノルムを定義

$$L = \frac{1}{2}\left(\dot{\boldsymbol{\phi}}^{\mathrm{T}} - \boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{f}^{\mathrm{T}} - \boldsymbol{\varphi}^{\mathrm{T}}\tilde{\boldsymbol{f}}^{\mathrm{T}}\right)\boldsymbol{G}\left(\dot{\boldsymbol{\phi}} - \boldsymbol{f}\boldsymbol{\phi} - \tilde{\boldsymbol{f}}\boldsymbol{\varphi}\right) - \frac{1}{2}\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{V}\boldsymbol{\phi} - \boldsymbol{\phi}^{\mathrm{T}}M\boldsymbol{\varphi} - \frac{1}{2}\boldsymbol{\varphi}^{\mathrm{T}}\tilde{\boldsymbol{V}}\boldsymbol{\varphi}$$

$$(\boldsymbol{\phi}_{\mathcal{N}}, \boldsymbol{\phi}_{\mathcal{M}})_{\mathrm{KG}} = -i\left\{\boldsymbol{\phi}_{\mathcal{N}}^{\mathrm{T}}\boldsymbol{G}\left(\overline{\dot{\boldsymbol{\phi}}_{\mathcal{M}}} - \boldsymbol{f}\overline{\boldsymbol{\phi}_{\mathcal{M}}} - \tilde{\boldsymbol{f}}\overline{\boldsymbol{\varphi}_{\mathcal{M}}}\right) - \left(\dot{\boldsymbol{\phi}}_{\mathcal{N}}^{\mathrm{T}} - \boldsymbol{\phi}_{\mathcal{N}}^{\mathrm{T}}\boldsymbol{f}^{\mathrm{T}} - \boldsymbol{\varphi}_{\mathcal{N}}^{\mathrm{T}}\tilde{\boldsymbol{f}}^{\mathrm{T}}\right)\boldsymbol{G}\overline{\boldsymbol{\phi}_{\mathcal{M}}}\right\}$$

補助場を含む一般的な作用からKGノルムを定義する方法を開発［困難3を回避］

チェック項目3:規格化

➢(例) η成分 $$A_\eta^{\ell m}(\eta, r) = \sum_p \frac{\chi_p(\eta)}{a(\eta)} f^{p\ell}(r)$$

$p^2 \Leftrightarrow (kr_{curv})^2$ -2に対応

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}\eta^2} + m_A^2 a^2\right]\chi_p = p^2 \chi_p$$

✓ 有効ポテンシャルが常に正

✓ $a A_\eta \to e^{-|\eta|}$ @ $|\eta| \to \infty$

⇨ p2<0の解、つまり超曲率モードは存在しない!

まとめ

1. 2次の作用を物理的自由度のみで書き下したか

時空の対称性が低いにもかかわらず、スカラー/ベクトルモードが作用レベルで完全に分離

2. Klein-Gordonノルムは適切に定義したか

補助場を含む一般的な作用からKGノルムを定義する方法

3. モードは規格化可能になっているか

⇨ ゲージ場に超曲率モードは存在しない

本当に正しいの？

✓ [Frob+Higuchi (2014)] (Poincare座標)

$$\lim_{m_A \to 0} \langle 0 | A_\mu(x) A_{\mu'}(x') | 0 \rangle = \lim_{m_A \to 0} \frac{1}{m_A^2} \partial_\mu \partial_{\mu'} \Delta_{M^2}(Z(x, x')) + \mathcal{O}(m_A^0)$$

ここで $$\Delta_{M^2}(Z) = \frac{H^2}{(4\pi)^2} \frac{\Gamma(3+\nu')\Gamma(-\nu')}{\Gamma(2)} {}_2F_1\left(3+\nu', -\nu'; 2; \frac{1+Z}{2}\right)$$

⇕ 一致

✓ 我々の計算 (open-chart)

$$\lim_{m_A \to 0} \langle 0 | A_\eta(x) A_{\eta'}(x') | 0 \rangle = \lim_{m_A \to 0} \sum_{\sigma = \pm} \sum_{p\ell m} A_{\eta, \sigma p \ell m}(\eta_{\mathrm{J}}, r_{\mathrm{J}}, \Omega) \overline{A_{\eta, \sigma p \ell m}(\eta'_{\mathrm{J}}, r'_{\mathrm{J}}, \Omega')}$$