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Breaking of Vainshtein screening in scalar-tensor theories beyond Horndeski

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Reference

Breaking of Vainshtein screening in scalar-tensor theories beyond Horndeski
Tsutomu Kobayashi(Rikkyo), Yuki Watanabe(RESCEU) and **DY**, arXiv:1411.4130.

Why modified gravity?

- Discovery of cosmic acceleration

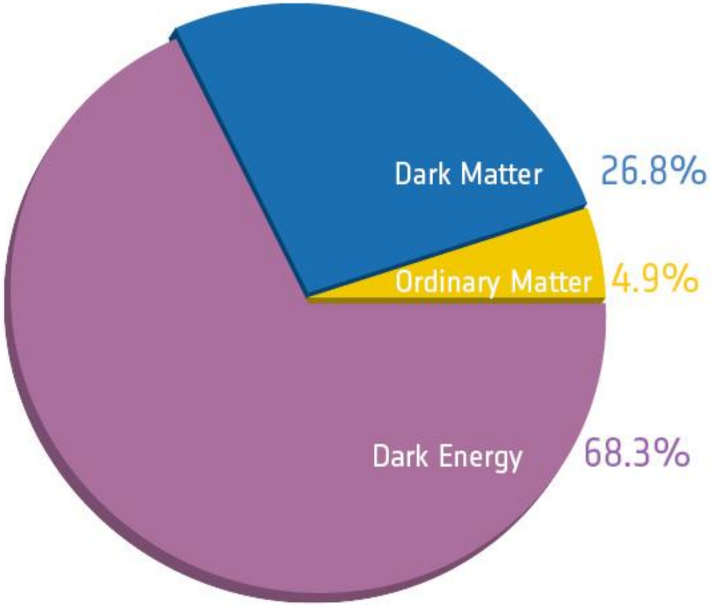
  : Our understanding of the Universe is incomplete!

- Need better understanding of gravity:

  -- Dark energy or modified gravity ?

  - Small scale : consistent with experiments in the solar-system and on the Earth
  - Large scale : can be tested though the cosmological observations

[Planck2013]



Modified gravity theories

- General relativity : massless spin-2

- Modified gravity : A new d.o.f. is introduced to achieve the accelerating expansion

  $f(R)$, DGP, Galileon, massive gravity, bi-gravity, …

✓ Scalar-tensor theories

- Gravity is mediated by $g_{\mu\nu}$ and ϕ
- Can capture essential modification of gravity

Modified gravity theories

- Scalars mediates fifth forces strongly constrained by precision tests of gravity at Solar system scale:

$$\text{E.g.} \quad |\Psi/\Phi - 1| < O(10^{-4})$$

$\Rightarrow$ Scalar field must be screened in the vicinity of sources.

- *Need screening mechanism on short scales*

- Cosmological test of gravity theories

VAINSHTEIN SCREENING MECHANISM

$$ds^2 = -\left(1 + 2\Phi(\boldsymbol{x})\right)dt^2 + \left(1 - 2\Psi(\boldsymbol{x})\right)d\boldsymbol{x}^2$$

✓General Relativity

$$\nabla\Phi = \nabla\Psi = G_N M/r^2$$

☺

✓Scalar-tensor theories

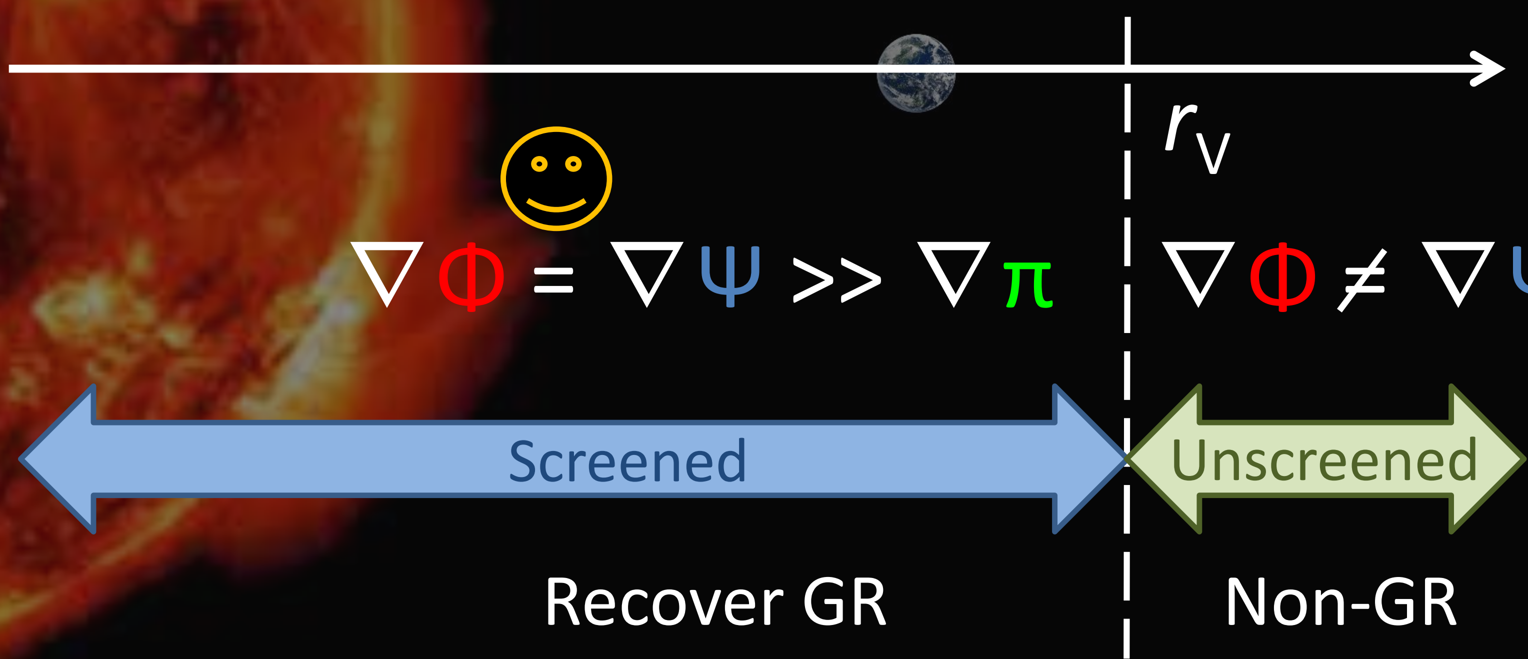
Scalar d.o.f. can participate in gravity

dynamics : $\phi(x) = \phi_0 + \pi(x)$

⇒ $\nabla\Phi \neq \nabla\Psi$!!!

$$\mathrm{d}s^2 = -\left(1 + 2\Phi(\boldsymbol{x})\right)\mathrm{d}t^2 + \left(1 - 2\Psi(\boldsymbol{x})\right)\mathrm{d}\boldsymbol{x}^2$$

Any modified model involving scalar dof should accommodate a mechanism to suppress the scalar interaction on small scales



An example : cubic Galileon

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 + \frac{1}{4\Lambda^3}(\partial\pi)^2\Box\pi - \frac{1}{M_{\rm Pl}}\pi\rho$$

✓ Look for spherical symmetric solution : ρ=ρ(r), π = π(r)

Algebraic equation for π'/Λ³r

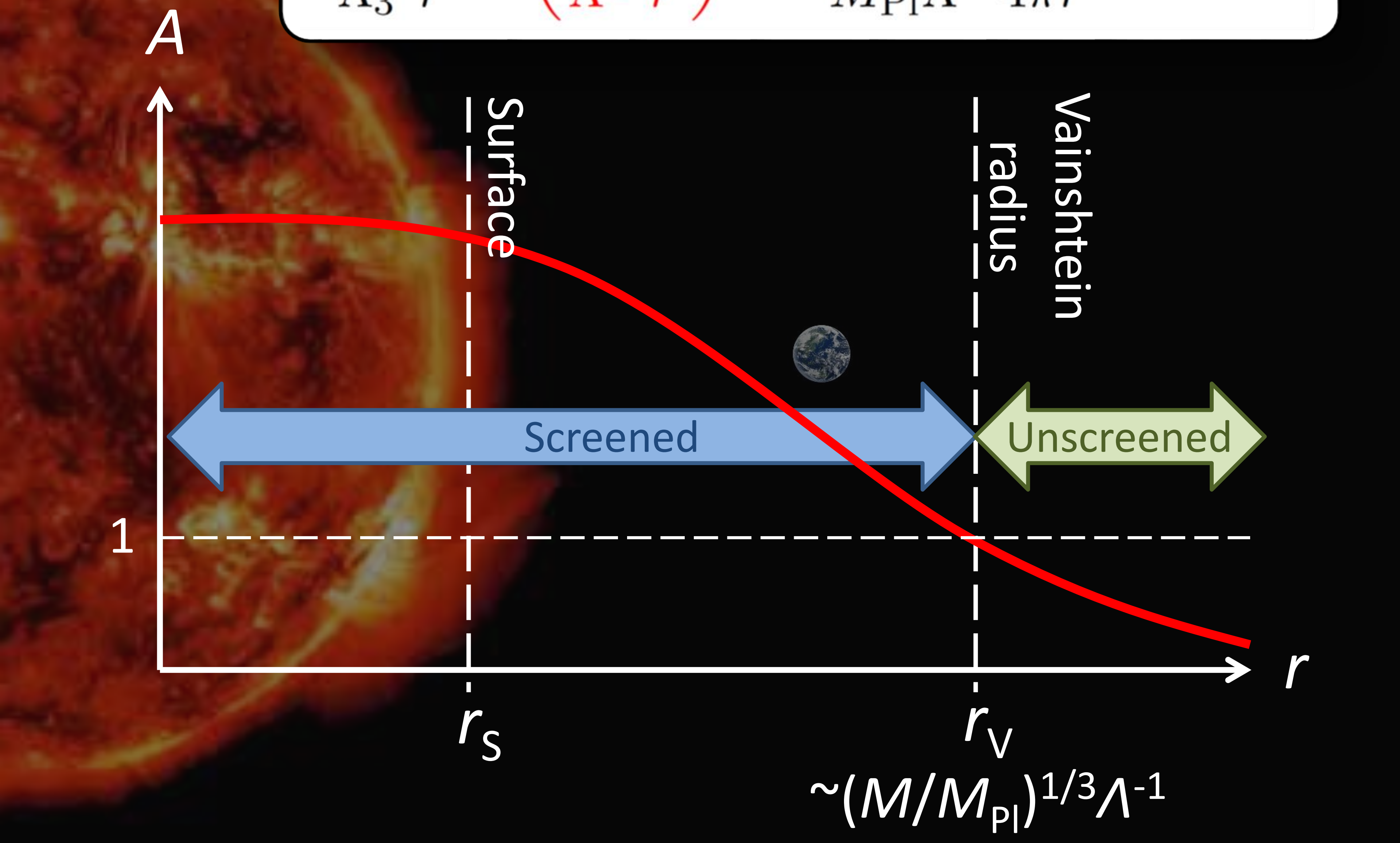
Enclosed mass ∝ ρ

$$\frac{1}{\Lambda_3}\frac{\pi'}{r} + \left(\frac{1}{\Lambda^3}\frac{\pi'}{r}\right)^2 = \frac{1}{M_{\rm Pl}\Lambda^3}\frac{M(r)}{4\pi r^3} \equiv A(r)$$

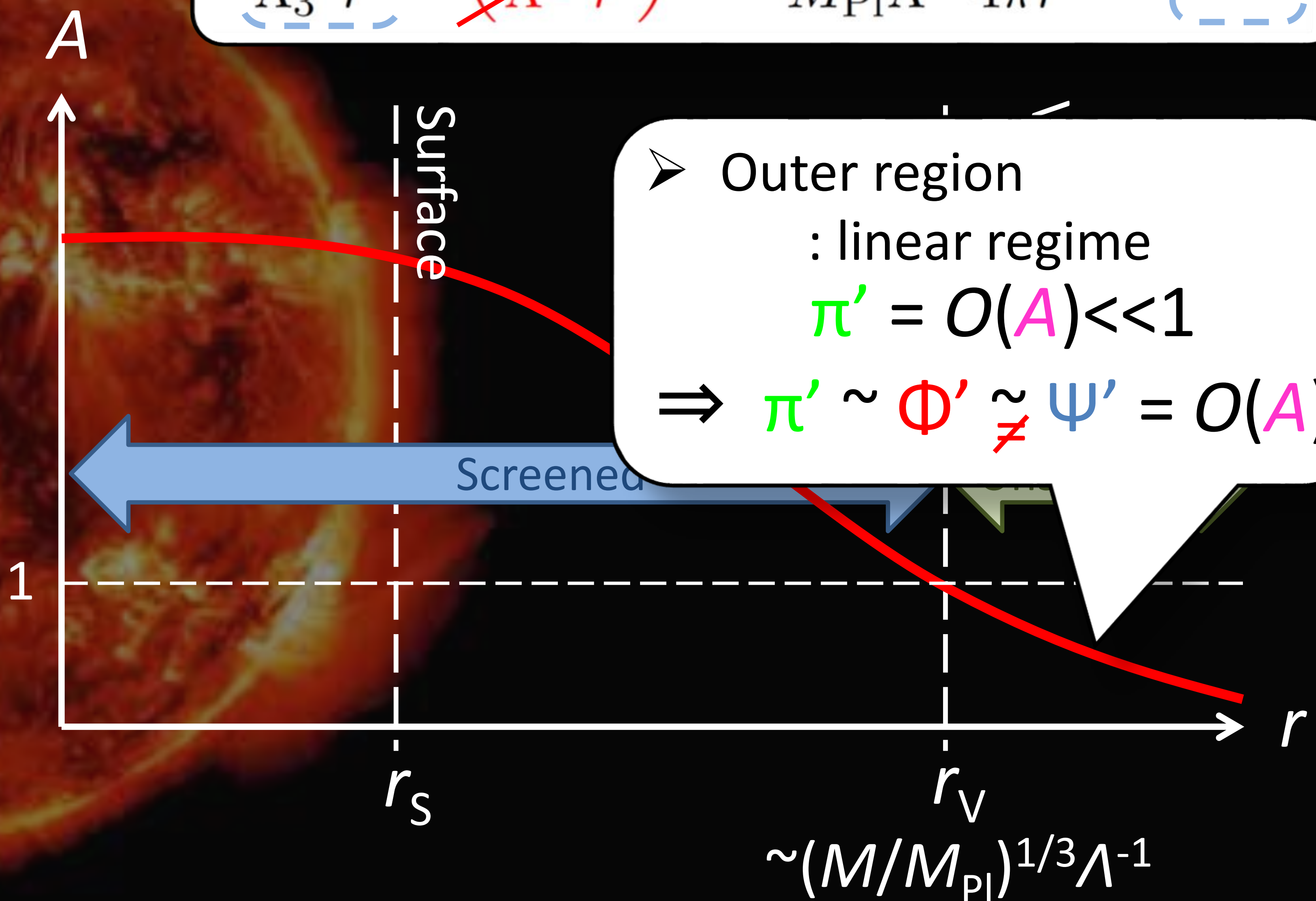
∝ A(r)

$$\Rightarrow \begin{cases} M_{\rm Pl}\Phi' = -\pi' + \dfrac{M(r)}{8\pi M_{\rm Pl} r^2} \\ M_{\rm Pl}\Psi' = +\pi' + \dfrac{M(r)}{8\pi M_{\rm Pl} r^2} \end{cases}$$

$$\frac{1}{\Lambda_3}\frac{\pi'}{r} + \left(\frac{1}{\Lambda^3}\frac{\pi'}{r}\right)^2 = \frac{1}{M_{\mathrm{Pl}}\Lambda^3}\frac{M(r)}{4\pi r^3} \equiv A(r)$$



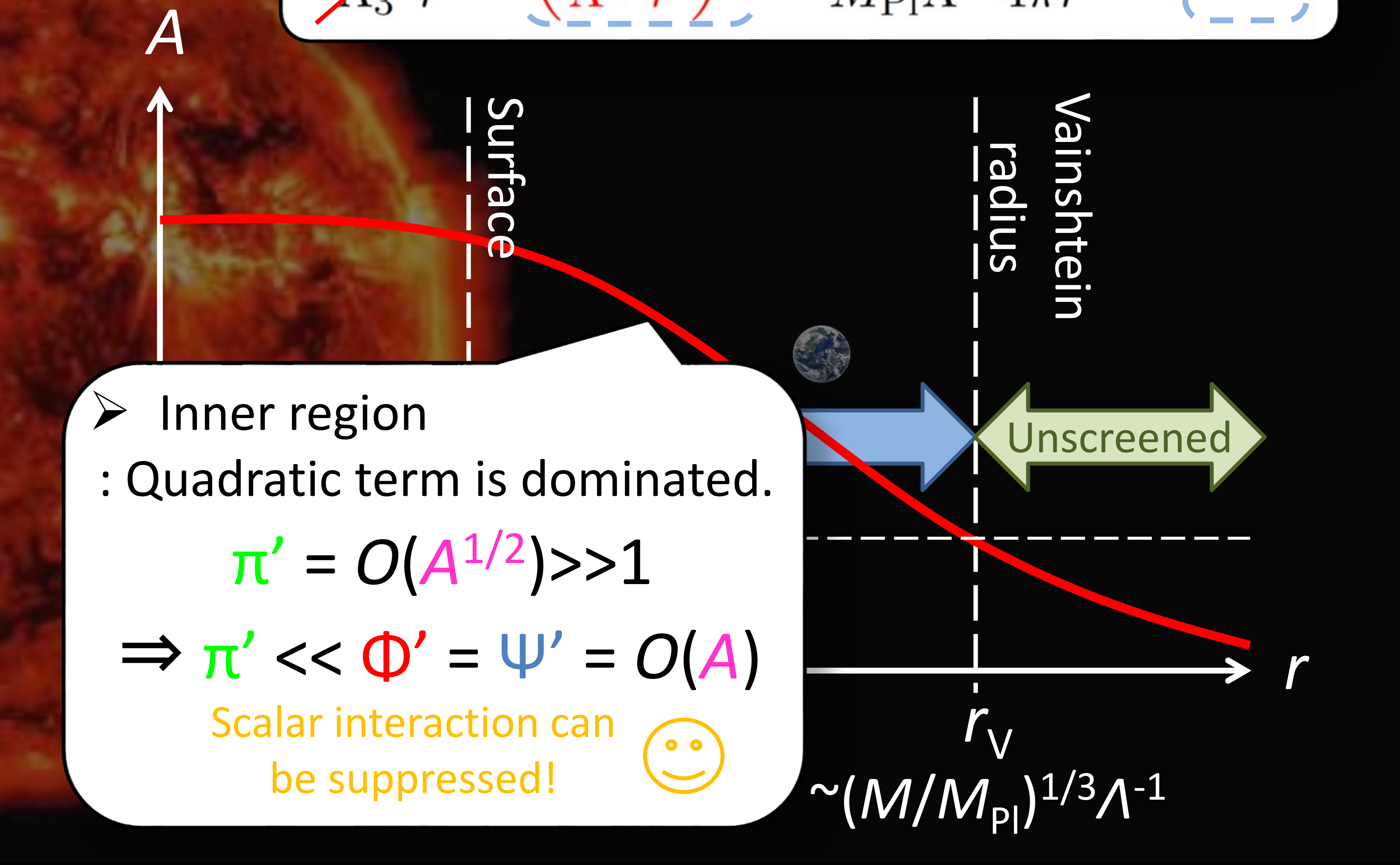
$$\boxed{\frac{1}{\Lambda_3}\frac{\pi'}{r}} + \cancel{\left(\frac{1}{\Lambda^3}\frac{\pi'}{r}\right)^2} = \frac{1}{M_{\rm Pl}\Lambda^3}\frac{M(r)}{4\pi r^3} \equiv \boxed{A(r)}$$



- Outer region
  : linear regime
  $\pi' = O(A) \ll 1$

$\Rightarrow \pi' \sim \Phi' \gneqq \Psi' = O(A)$

$$\cancel{\frac{1}{\Lambda_3}\frac{\pi'}{r}} + \boxed{\left(\frac{1}{\Lambda^3}\frac{\pi'}{r}\right)^2} = \frac{1}{M_{\mathrm{Pl}}\Lambda^3}\frac{M(r)}{4\pi r^3} \equiv \boxed{A(r)}$$



- Inner region

: Quadratic term is dominated.

$$\pi' = O(A^{1/2}) >> 1$$

$$\Rightarrow \pi' << \Phi' = \Psi' = O(A)$$

Scalar interaction can be suppressed! ☺

Lesson from this example

- Naively, scalar-tensor theories predict

$$\nabla \pi \sim \nabla \Phi \underset{\neq}{\sim} \nabla \Psi$$

- Nonlinear derivative interaction of π can be large, leading to self-screening in the vicinity of source (even if $\Phi \ll 1$)

$$(\nabla^2 \pi)^n \nabla \pi \sim \nabla \pi \ll \nabla \Phi = \nabla \Psi$$

  - ✓ 2 derivatives acting on π plays central role for implementing Vainshtein mech.

EFFECTIVE THEORY FOR VAINSHTEIN MECHANISM

[Horndeski (1974),Deffayet+Gao+Steer+Zahariade (2011), Kobayashi+Yamaguchi+Yokoyama (2011)]

Horndeski theory

: The most general scalar-tensor theory with second-order field equations.

$$
\begin{aligned}
\mathcal{L} =& G_2(\phi, X) - G_3(\phi, X)\Box\phi \\
&+ G_4(\phi, X)R + \frac{\partial G_4}{\partial X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\
&+ G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\
&- \frac{1}{6}\frac{\partial G_5}{\partial X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}
$$

✓have **4** arbitrary functions of ϕ and $X=-1/2(\nabla\phi)^2$.

✓is quite useful for a comprehensive study of modified gravity.

Effective theory for screening mech.

[Cosmological background :
Kimura+Kobayashi+Yamamoto(2012), Kobayashi+Watanabe+**DY** (2014)]
[Minkowski background :
Narikawa+Kobayashi+**DY**+Saito (2013), Koyama+Niz+Tasinato(2013)]

The general theory exhibiting Vainshtein screening mechanism can be derived from Horndeski Lagrangian.

Step 1 : Expand the Lagrangian around FLRW background

$$\mathrm{d}s^2 = -\left(1 + 2\,\epsilon \frac{\Phi(t, \boldsymbol{x})}{M_{\mathrm{Pl}}}\right) \mathrm{d}t^2 + a^2(t) \left(1 - 2\,\epsilon \frac{\Psi(t, \boldsymbol{x})}{M_{\mathrm{Pl}}}\right) \mathrm{d}\boldsymbol{x}^2$$

$$\phi = \phi_0(t) + \epsilon\pi(t, \boldsymbol{x})$$

Effective theory for screening mech.

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Kimura+Kobayashi+Yamamoto(2012), Kobayashi+Watanabe+**DY** (2014)]
[Minkowski background :
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Step 2 : Quasi-static approximation

$$(\nabla\epsilon)^2 \gg (\partial_t\epsilon)^2 \sim \max\left[H^2\epsilon^2, M^2\epsilon^2\right]$$

Step 3 : Assume scaling of coefficients

$$G_{3X} = \mathcal{O}\left(\Lambda^{-3}\right) \quad G_{4XX} = \mathcal{O}\left(\Lambda^{-6}\right)$$ etc...

Note : A concrete realization of the above scaling can be found in massive gravity.

Effective theory for screening mech.

[Cosmological background :
Kimura+Kobayashi+Yamamoto(2012), Kobayashi+Watanabe+**DY** (2014)]
[Minkowski background :
Narikawa+Kobayashi+**DY**+Saito (2013), Koyama+Niz+Tasinato(2013)]

Step 4 : Keep (2+*n*)-th order terms with (2+2*n*) spatial derivatives

$$\left(\nabla^2\epsilon\right)^n\left(\nabla\epsilon\right)^2 \Longleftrightarrow \nabla^{2n+2}\epsilon^{2+n}$$

- These can be larger than the quadratic terms below a certain scale.
- Such nonlinear derivative interactions are crucial for realizing the Vainshtein mechanism.

Effective theory from Horndeski

[Kimura+Kobayashi+Yamamoto(2012), Kobayashi+Watanabe+**DY** (2014)]

$$
\begin{aligned}
\mathcal{L}^{\rm eff}_{\rm Horndeski} = & - a\mathcal{F}\Psi\nabla^2\Psi + 2a\mathcal{G}\Psi\nabla^2\Phi + \frac{a\eta}{2}\pi\nabla^2\pi \\
& - a\xi_1\Phi\nabla^2\pi + 4a\xi_2\Psi\nabla^2\pi - \frac{a^3\rho}{M_{\rm Pl}}\Phi\delta \\
& + \frac{\mu}{a\Lambda^3}\mathcal{L}^{\rm Gal}_3 + \frac{\nu}{a^3\Lambda^6}\mathcal{L}^{\rm Gal}_4 \\
& - \frac{\alpha_1}{a\Lambda^3}\Phi\mathcal{E}^{\rm Gal}_3 + \frac{\alpha_2}{a\Lambda^3}\Psi\mathcal{E}^{\rm Gal}_3 - \frac{\beta}{a^3\Lambda^6}\Phi\mathcal{E}^{\rm Gal}_4 \\
& + \frac{\gamma}{a\Lambda^3}\nabla^i\Phi\nabla^j\Psi\left(\delta_{ij}\nabla^2\pi - \nabla_i\nabla_j\pi\right)
\end{aligned}
$$

Effective theory from Horndeski

[Kimura+Kobayashi+Yamamoto(2012), Kobayashi+Watanabe+**DY** (2014)]

$$\mathcal{L}^{\rm eff}_{\rm Horndeski} = -a\mathcal{F}\Psi\nabla^2\Psi + 2a\mathcal{G}\Psi\nabla^2\Phi + \frac{a\eta}{2}\pi\nabla^2\pi - a\xi_1\Phi\nabla^2\pi + 4a\xi_2\Psi\nabla^2\pi - \frac{a^3\rho}{M_{\rm Pl}}\Phi\delta$$

- ✓ $O(\nabla^2\varepsilon^2)$ terms
- ✓ *F*, *G*, *η*,… are O(1) unless they vanish.
- ✓ Matter overdensity δ is assumed to be $O(\nabla^2\Phi)$.

$\mathcal{E}_4^{\rm Gal}$

Effe

$$\mathcal{L}^{\rm eff}_{\rm Horn}$$

✓ $O((\nabla^2\varepsilon)^n(\nabla\varepsilon)^2)$ terms

$$\mathcal{L}_3^{\rm Gal} = -\frac{1}{2}(\nabla\pi)^2(\nabla^2\pi)$$

$$\mathcal{L}_4^{\rm Gal} = -\frac{1}{2}(\nabla\pi)^2\left\{(\nabla^2\pi)^2 - (\nabla_i\nabla_j\pi)^2\right\}$$

$$\mathcal{E}_3^{\rm Gal} = (\nabla^2\pi)^2 - (\nabla_i\nabla_j\pi)^2$$

$$\mathcal{E}_4^{\rm Gal} = (\nabla^2\pi)^3 - 3(\nabla^2\pi)(\nabla_i\nabla_j\pi)^2 + 2(\nabla_i\nabla_j\pi)^3$$

$$+\frac{\mu}{a\Lambda^3}\mathcal{L}_3^{\rm Gal} + \frac{\nu}{a^3\Lambda^6}\mathcal{L}_4^{\rm Gal}$$

$$-\frac{\alpha_1}{a\Lambda^3}\Phi\mathcal{E}_3^{\rm Gal} + \frac{\alpha_2}{a\Lambda^3}\Psi\mathcal{E}_3^{\rm Gal} - \frac{\beta}{a^3\Lambda^6}\Phi\mathcal{E}_4^{\rm Gal}$$

$$+\frac{\gamma}{a\Lambda^3}\nabla^i\Phi\nabla^j\Psi\left(\delta_{ij}\nabla^2\pi - \nabla_i\nabla_j\pi\right)$$

Effective theory from Horndeski

✓Hereafter we focus on the subclass of β=γ=0, which amounts for switching off G_5

- β≠0 → Vainshtein solutions are unstable [Koyama+(2013)]
- γ≠0 → Metric potentials do not show the correct Newtonian behavior [Kobayashi+(2013)]

$$\mathcal{L}^{\rm eff}_{\rm Hor\ldots}$$

$$+\frac{\mu}{a\Lambda^3}\mathcal{L}_3^{\rm Gal}+\frac{\nu}{a^3\Lambda^6}\mathcal{L}_4^{\rm Gal}$$

$$-\frac{\alpha_1}{a\Lambda^3}\Phi\mathcal{E}_3^{\rm Gal}+\frac{\alpha_2}{a\Lambda^3}\Psi\mathcal{E}_3^{\rm Gal}\cancel{-\frac{\beta}{a^3\Lambda^6}\Phi\mathcal{E}_4^{\rm Gal}}$$

$$\cancel{+\frac{\gamma}{a\Lambda^3}\nabla^i\Phi\nabla^j\Psi\left(\delta_{ij}\nabla^2\pi-\nabla_i\nabla_j\pi\right)}$$

Spherical overdensity from Horndeski

In terms of $x = \frac{1}{\Lambda^3}\frac{\pi'}{a^2 r}$ $\quad A = \frac{1}{M_{\rm Pl}\Lambda^3}\frac{M(t,r)}{8\pi r^3}$

Algebraic equation for *x* can be written as

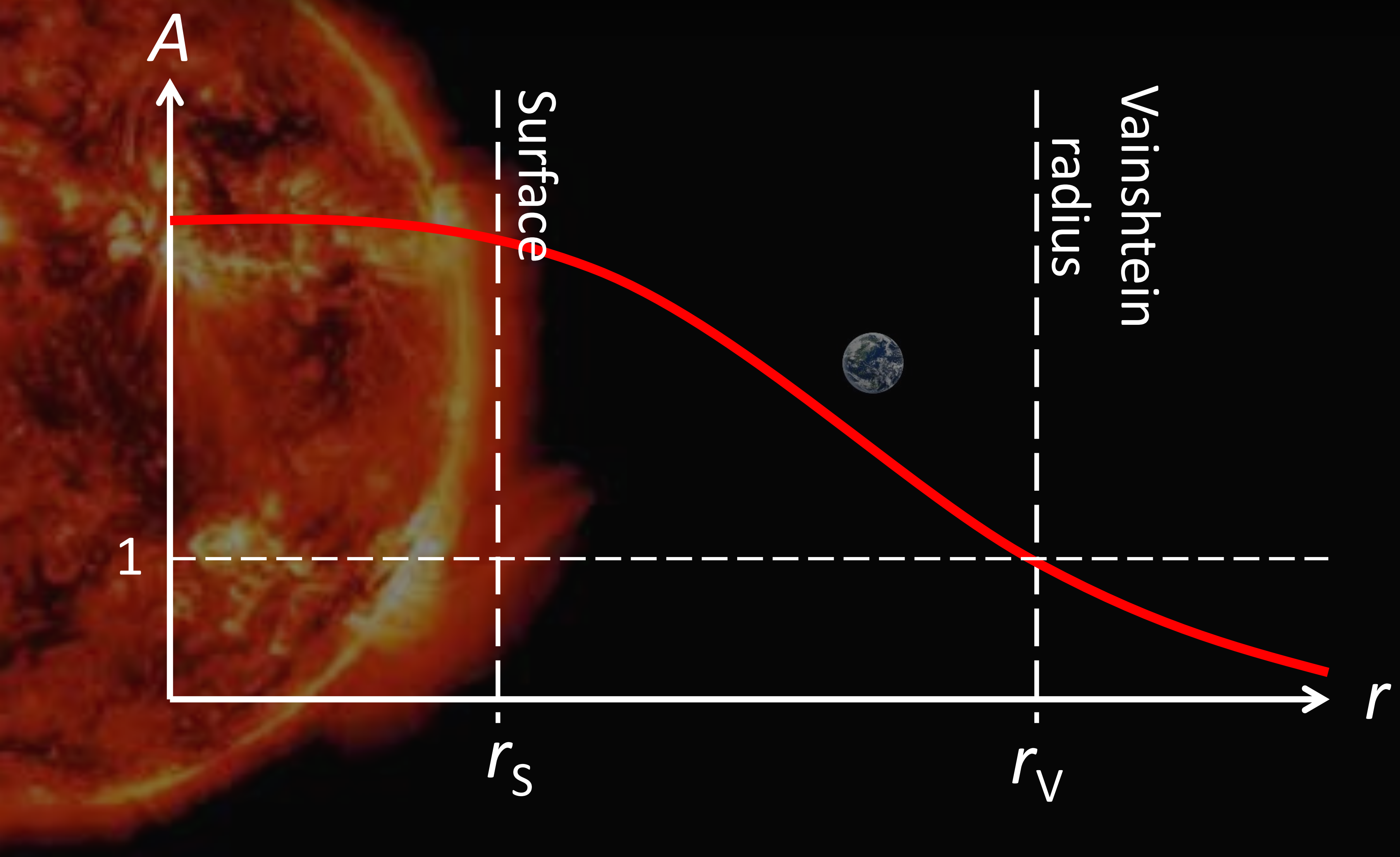
$$\mathcal{O}(1)A + 2\Big[\mathcal{O}(1)+\mathcal{O}(1)A\Big]x + \mathcal{O}(1)x^2 - \mathcal{O}(1)x^3 = 0$$

$$\Downarrow$$

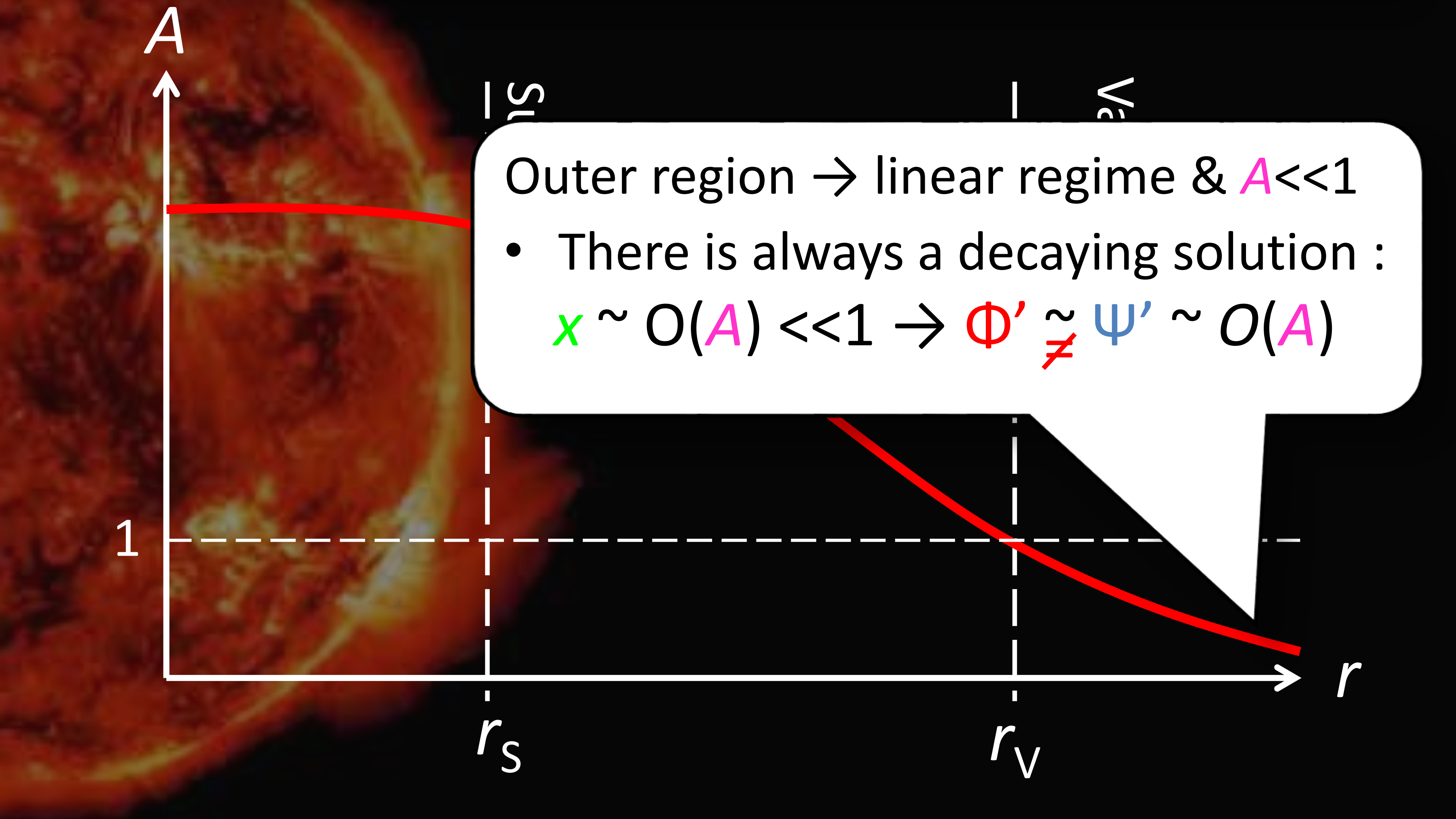
$$x = x[A] \Longrightarrow \begin{cases} \dfrac{1}{\Lambda^3}\dfrac{\Phi'}{r} = \mathcal{O}(1)x + \mathcal{O}(1)x^2 + \mathcal{O}(1)A \\ \dfrac{1}{\Lambda^3}\dfrac{\Psi'}{r} = \mathcal{O}(1)x + \mathcal{O}(1)x^2 + \mathcal{O}(1)A \end{cases}$$

Solving algebraic equation for x is essential
for constructing Vainshtein solution.

$$\mathcal{O}(1)A + 2\Big[\mathcal{O}(1) + \mathcal{O}(1)A\Big]x + \mathcal{O}(1)x^2 - \mathcal{O}(1)x^3 = 0$$



$$\boxed{\mathcal{O}(1)A} + 2\left[\boxed{\mathcal{O}(1)} + \cancel{\mathcal{O}(1)A}\right]x + \cancel{\mathcal{O}(1)x^2} - \cancel{\mathcal{O}(1)x^3} = 0$$



Outer region → linear regime & $A \ll 1$

- There is always a decaying solution :

  $x \sim O(A) \ll 1 \rightarrow \Phi' \not\simeq \Psi' \sim O(A)$

$$\mathcal{O}(1)A + 2\left[\cancel{\mathcal{O}(1)} + \boxed{\mathcal{O}(1)A\,]\,x}\right. + \cancel{\mathcal{O}(1)x^2} - \boxed{\mathcal{O}(1)x^3} = 0$$

Inner region → nonlinear regime & A>>1

- Imposing the condition that two of three terms balance:

✓ $x \sim O(A^{1/2})$ case

→ $\Phi'/a^2 M_{Pl} = \Psi'/a^2 M_{Pl} = G_N(t) M/r^2$

GR can be reproduced inside the Vainshtein radius ☺

✓ (other cases → $\Phi' \neq \Psi'$ even in $r < r_V$)

EFFECTIVE THEORY FROM BEYOND-HORNDESKI

Horndeski theory

can be generalized w/o introducing any propagating dof : *beyond-Horndeski theory*.

[Gleyzes+Langlois+Piazza+Vernizzi (2014)]

- ✓Inevitably has higher-order field equations.
- ✓Introduces a preferred time-slicing on which # of initial conditions remain the same.

➢ Lagrangian in a covariant form by reviving ϕ:

$$\mathcal{L} = \mathcal{L}_{\mathrm{Horndeski}} + \mathcal{L}_{\mathrm{beyond}}$$

Horndeski theory

can be generalized w/o introducing any

$$\mathcal{L}_{\rm beyond} = F_4(\phi, X)\Big\{\nabla^\mu\phi\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi\Box\phi - \nabla^\mu\nabla_\mu\nabla_\lambda\phi\nabla^\nu\nabla_\nu\nabla^\lambda\phi + X\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\Big\}$$

$$+ F_5(\phi, X)\Big\{(\Box\phi)^2\,\nabla^\mu\phi\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi - 2\Box\phi\nabla^\mu\phi\nabla_\mu\nabla_\lambda\phi\nabla^\nu\phi\nabla_\nu\nabla^\lambda\phi$$

$$- (\nabla_\mu\nabla_\nu\phi)^2\,\nabla^\rho\phi\nabla^\lambda\phi\nabla_\rho\nabla_\sigma\phi + 2\nabla^\mu\phi\nabla_\mu\nabla^\lambda\phi\nabla_\lambda\nabla^\rho\phi\nabla_\rho\nabla_\lambda\phi\nabla^\lambda\phi$$

$$+\frac{2}{3}X\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right]\Big\}$$

✓have **6 arbitrary functions** of ϕ and $X=-1/2(\nabla\phi)^2$.

➤ Lagrangian in a covariant form by rev

$$\mathcal{L} = \mathcal{L}_{\rm Horndeski} + \mathcal{L}_{\rm beyond}$$

[Kobayashi+Watanabe+**DY** (2014)]

New terms from beyond-Horndeski

$$\mathcal{L}^{\text{eff}} = \mathcal{L}^{\text{eff}}_{\text{Horndeski}} + \mathcal{L}^{\text{eff}}_{\text{beyond}}$$

➢ New terms

$$\xi_t\, ,\alpha_t\, ,\alpha_*\, ,\beta_* \propto X$$

$$\begin{aligned}\mathcal{L}^{\text{eff}}_{\text{beyond}} =& \frac{4a\xi_t}{\mathcal{M}}\dot{\Psi}\nabla^2\pi \\ &+ \frac{2\alpha_t}{a\Lambda^3\mathcal{M}}\dot{\Psi}\mathcal{E}_3^{\text{Gal}} - \frac{4\alpha_*}{a\Lambda^3}\nabla^i\Psi\nabla^j\pi\nabla_i\nabla_j\pi \\ &- \frac{4\beta_*}{a^3\Lambda^6}\nabla^i\Psi\nabla^j\pi\left[\left(\nabla^2\pi\right)\nabla_i\nabla_j\pi - \nabla_i\nabla^k\pi\nabla_k\nabla_j\pi\right]\end{aligned}$$

✓ Other terms generated by the beyond-Horndeski term can be absorbed in the redefinition of the corresponding coefficients.

✓ Taking limit $g_{\mu\nu}\to\eta_{\mu\nu}$, $X\to 0$, only the Honrdeski term survive and there is no new term.

[Kobayashi+Watanabe+**DY** (2014)]

New terms from beyond-Horndeski

$$\mathcal{L}^{\rm eff} = \mathcal{L}^{\rm eff}_{\rm Horndeski} + \mathcal{L}^{\rm eff}_{\rm beyond}$$

- New terms

$$\mathcal{L}^{\rm eff}_{\rm beyond} = \frac{4a\xi_t}{\mathcal{M}}\dot{\Psi}\nabla^2\pi + \frac{2\alpha_t}{a\Lambda^3\mathcal{M}}\dot{\Psi}\mathcal{E}_3^{\rm Gal} - \frac{4\alpha_*}{a\Lambda^3}\nabla^i\Psi\nabla^j\pi\nabla_i\nabla_j\pi \cancel{- \frac{4\beta_*}{a^3\Lambda^6}\nabla^i\Psi\nabla^j\pi\left[\left(\nabla^2\pi\right)\nabla_i\nabla_j\pi - \nabla_i\nabla^k\pi\nabla_k\nabla_j\pi\right]}$$

$$\xi_t\,,\,\alpha_t\,,\,\alpha_*\,,\,\beta_* \propto X$$

- ✓ Other terms generated by the beyond-Horndeski term can be absorbed in the redefinition of the corresponding coefficients.
- ✓ Taking limit $g_{\mu\nu}\to\eta_{\mu\nu}$, $X\to0$, only the Honrdeski term survive and there is no new term.

[Kobayashi+Watanabe+**DY** (2014)]

New terms from beyond-Horndeski

$$\mathcal{L}^{\rm eff} = \mathcal{L}^{\rm eff}_{\rm Horndeski} + \mathcal{L}^{\rm eff}_{\rm beyond}$$

- New terms

$$\mathcal{L}^{\rm eff}_{\rm beyond} = \boxed{\frac{4a\xi_t}{\mathcal{M}}\dot{\Psi}\nabla^2\pi} \boxed{+ \frac{2\alpha_t}{a\Lambda^3\mathcal{M}}\dot{\Psi}\mathcal{E}_3^{\rm Gal}} \boxed{- \frac{4\alpha_*}{a\Lambda^3}\nabla^i\Psi\nabla^j\pi\nabla_i\nabla_j\pi}$$
$$\boxed{\cancel{- \frac{4\beta_*}{a^3\Lambda^6}\nabla^i\Psi\nabla^j\pi\left[\left(\nabla^2\pi\right)\nabla_i\nabla_j\pi - \nabla_i\nabla^k\pi\nabla_k\nabla_j\pi\right]}}$$

EOM

$$\boxed{\nabla^2\partial_t\epsilon}$$

EOM

$$\boxed{\nabla^3\epsilon}$$

[Kobayashi+Watanabe+**DY** (2014)]

Impact of $\nabla^3\epsilon$: spherical overdensity

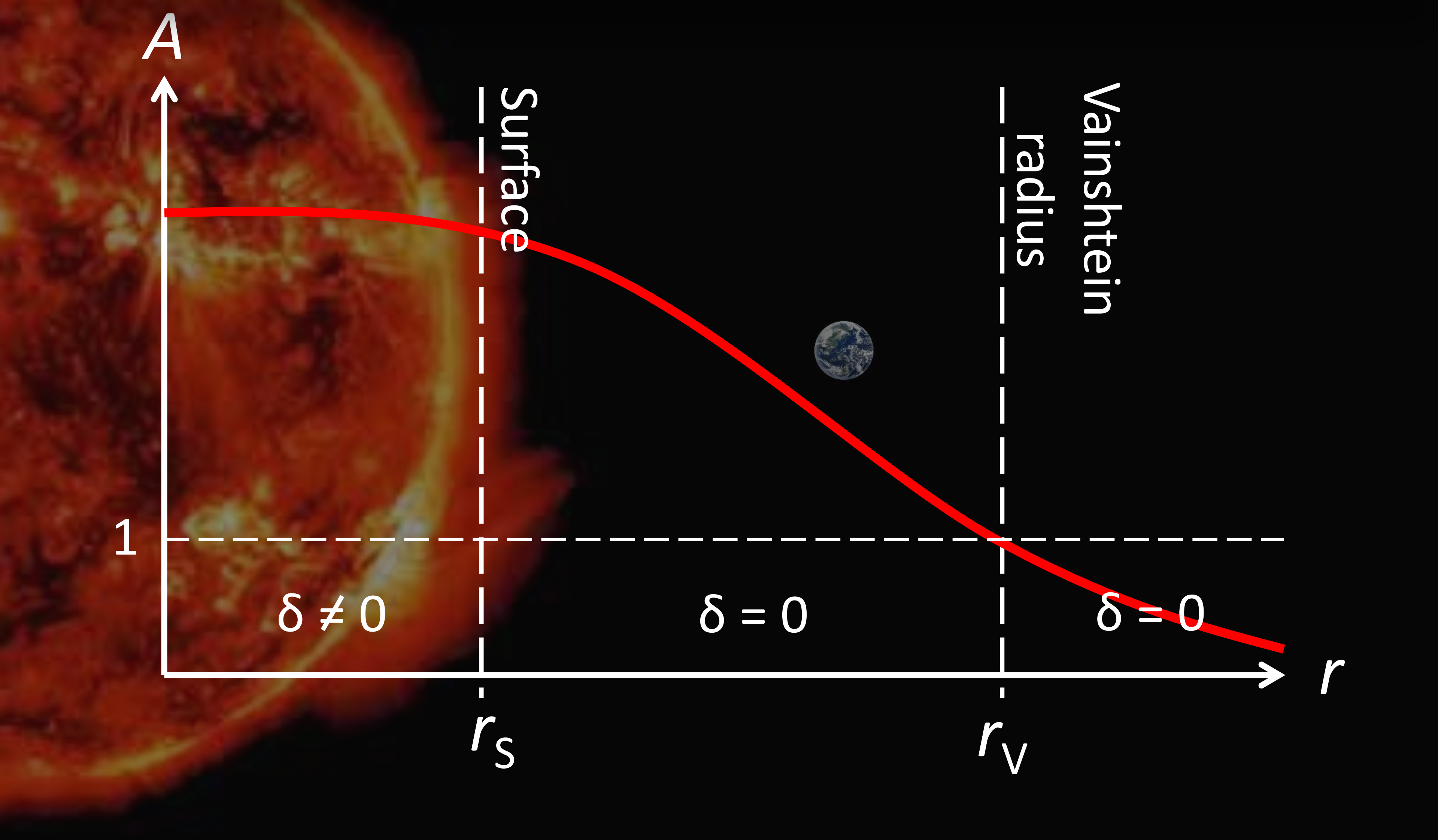
In terms of $x = \frac{1}{\Lambda^3}\frac{\pi'}{a^2 r}$ $\quad A = \frac{1}{M_{\rm Pl}\Lambda^3}\frac{M(t,r)}{8\pi r^3}$

Algebraic equation for *x* can be written as

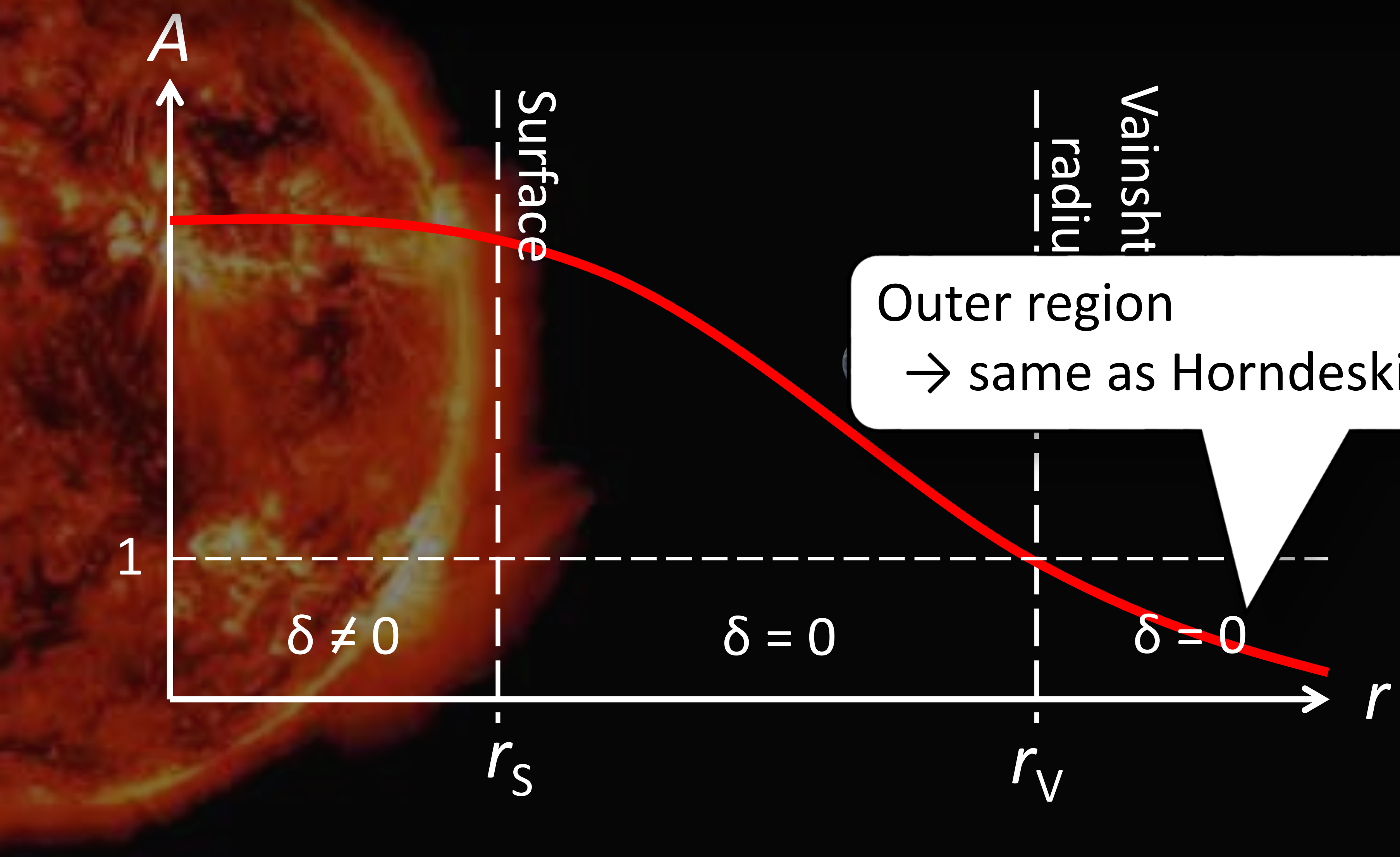
$$\mathcal{O}(1)A + 2\left[\mathcal{O}(1) + \mathcal{O}(1)A + \mathcal{O}(1)\alpha_* r A'\right]x + \mathcal{O}(1)x^2 - \mathcal{O}(1)x^3 = 0$$

Inner solution is determined not only the enclosed mass (M∝A) but also from the local energy density (A'~ρ)!

$$\mathcal{O}(1)A + 2\left[\mathcal{O}(1) + \mathcal{O}(1)A + \mathcal{O}(1)\alpha_* r A'\right]x + \mathcal{O}(1)x^2 - \mathcal{O}(1)x^3 = 0$$



$$\boxed{\mathcal{O}(1)A} + 2\left[\boxed{\mathcal{O}(1)} + \cancel{\mathcal{O}(1)A} + \cancel{\mathcal{O}(1)\alpha_* r A'}\right]x + \cancel{\mathcal{O}(1)x^2} - \cancel{\mathcal{O}(1)x^3} = 0$$



$$\mathcal{O}(1)A + 2\left[\cancel{\mathcal{O}(1)} + \mathcal{O}(1)A + \mathcal{O}(1)\alpha_* r A'\right]x + \cancel{\mathcal{O}(1)x^2} - \mathcal{O}(1)x^3 = 0$$

Inner region → nonlinear regime & *A, A', A''*>>1

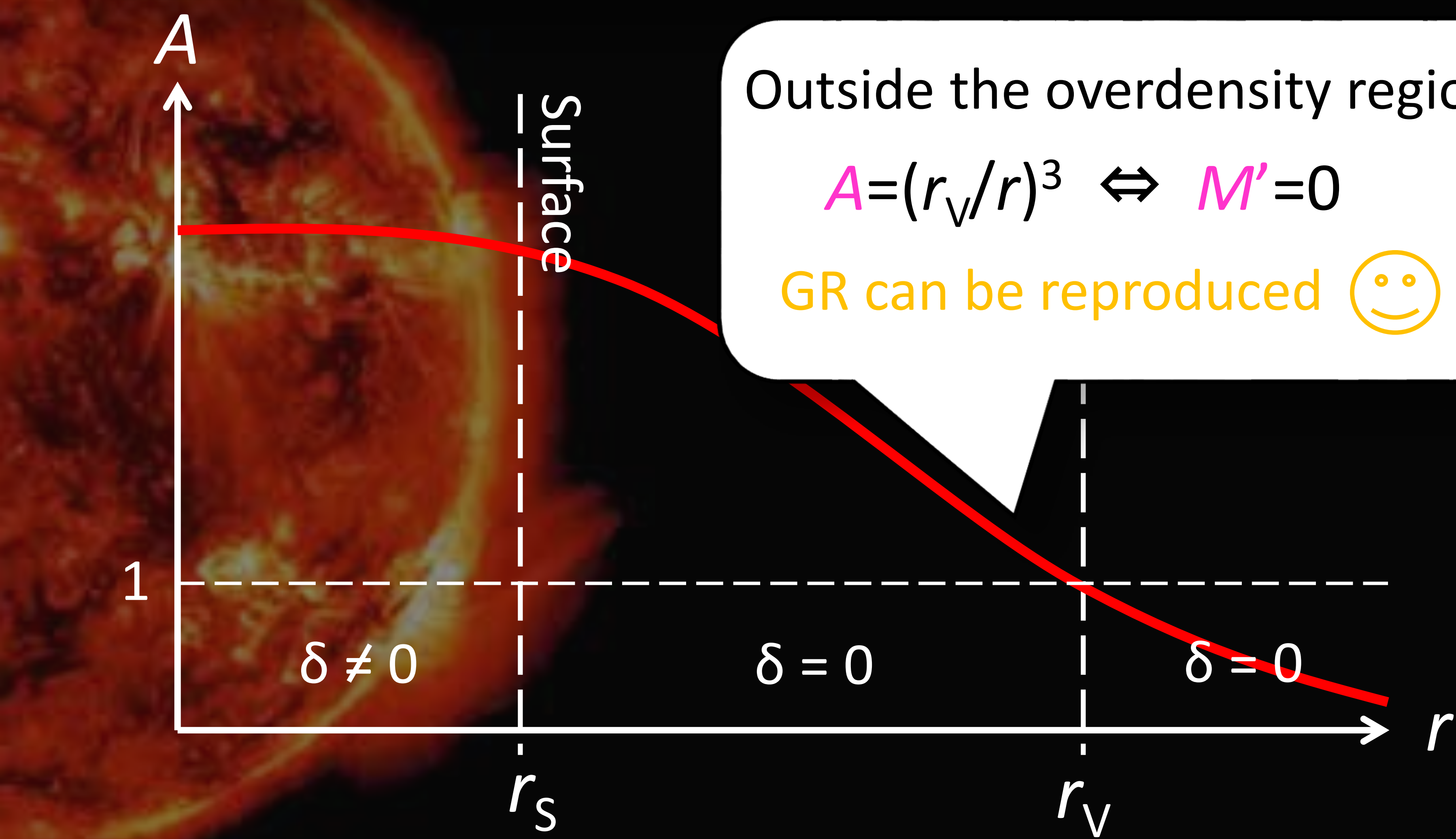
$$x^2 = \mathcal{O}(1)A + \mathcal{O}(1)\alpha_* r A'$$

Plugging the concrete expression of the coefficients,

$$\Longrightarrow \begin{cases} \dfrac{\Phi'}{a^2 M_{\rm Pl}} = \dfrac{G_{\rm N} M}{r^2} - \mathcal{O}(1)\alpha_*^2 \dfrac{M''}{8\pi M_{\rm Pl}^2} \\[2ex] \dfrac{\Psi'}{a^2 M_{\rm Pl}} = \dfrac{G_{\rm N} M}{r^2} - \mathcal{O}(1)\alpha_* \dfrac{M'}{8\pi M_{\rm Pl}^2 r} \end{cases}$$

New contribution

$$\mathcal{O}(1)A + 2\Big[\cancel{\mathcal{O}(1)} + \boxed{\mathcal{O}(1)A + \mathcal{O}(1)\alpha_* r A'\Big]x} + \cancel{\mathcal{O}(1)x^2} - \boxed{\mathcal{O}(1)x^3} = 0$$

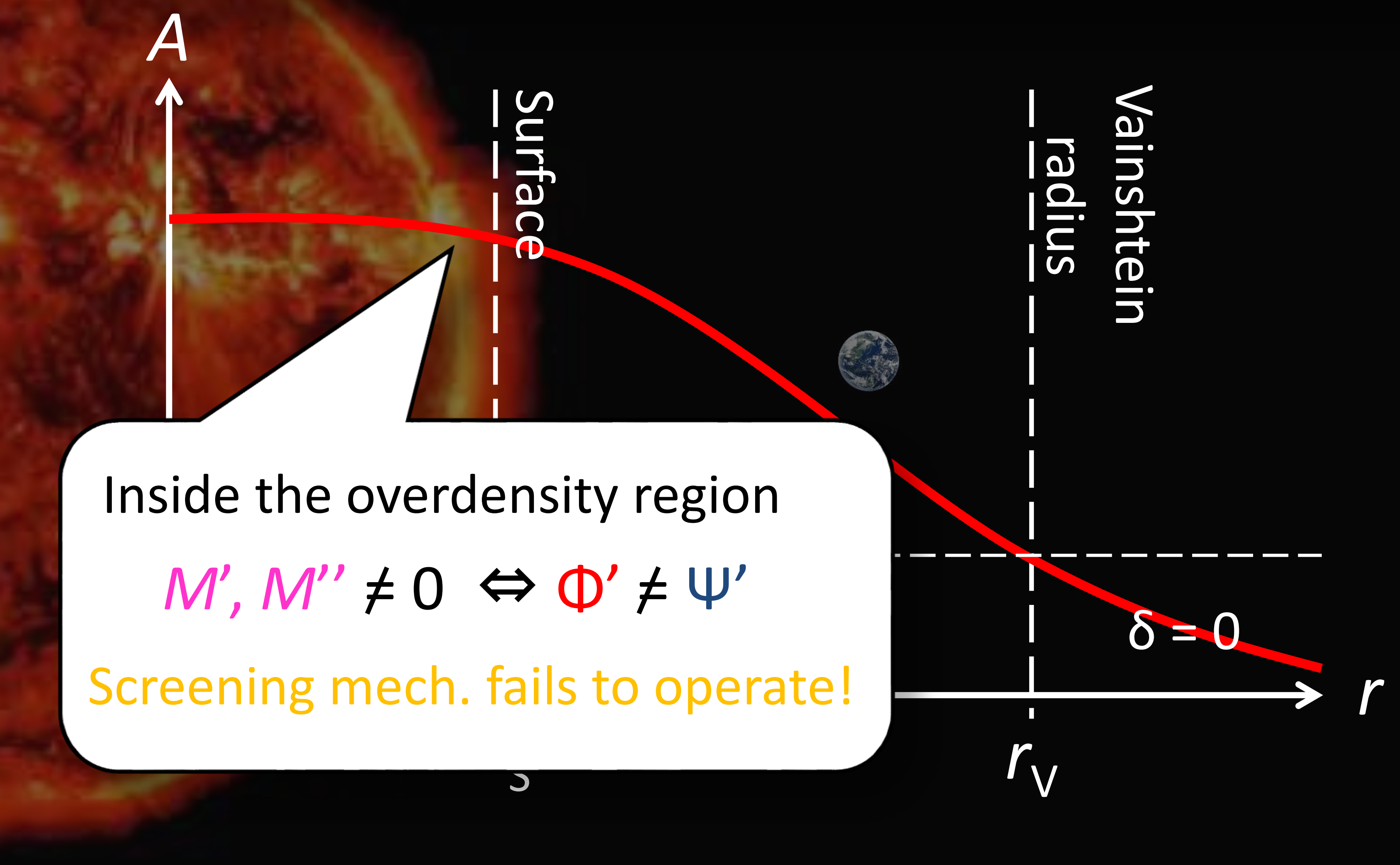


Outside the overdensity region

$A = (r_V/r)^3 \Leftrightarrow M' = 0$

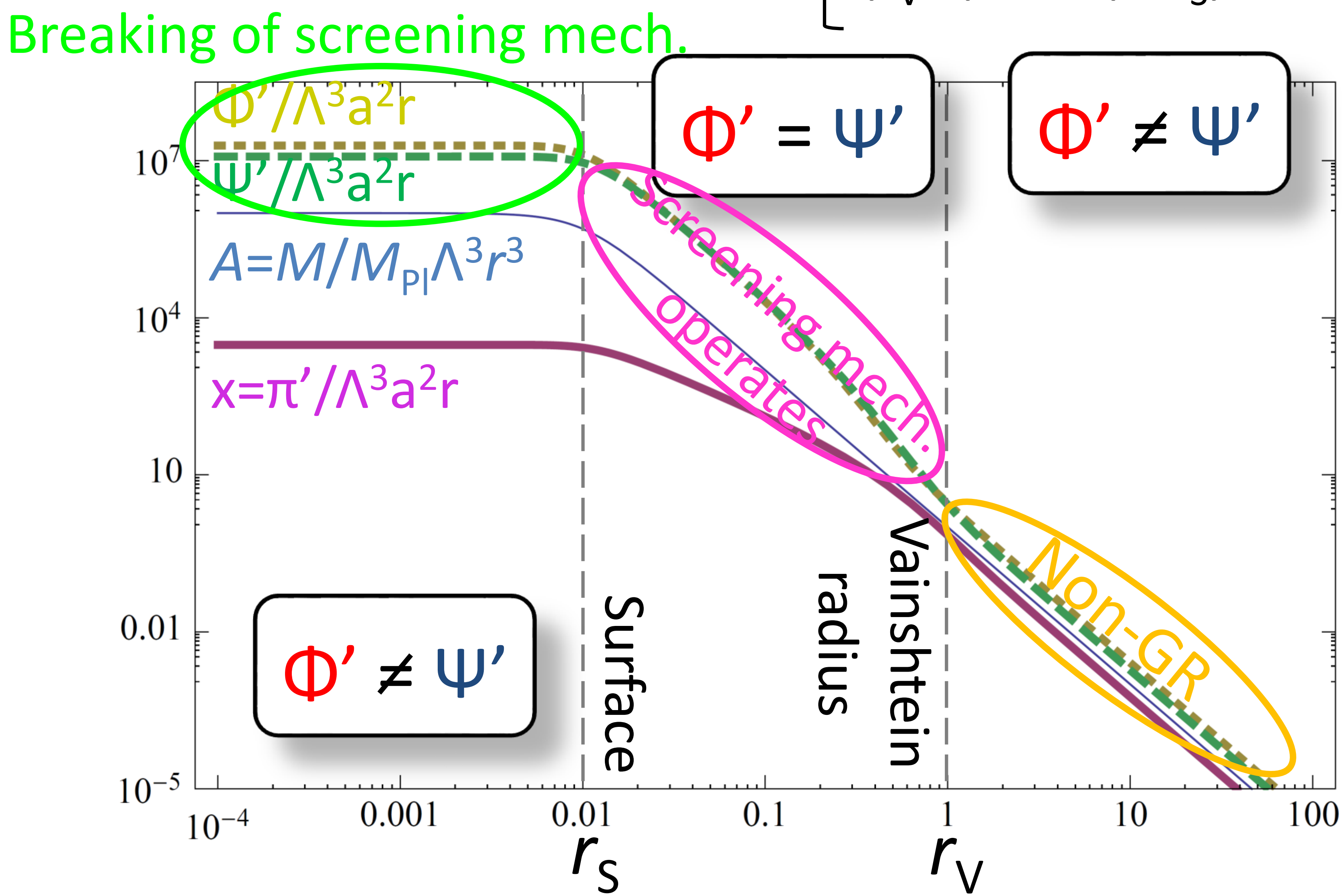
GR can be reproduced ☺

$$\mathcal{O}(1)A + 2\,\cancel{\left[\mathcal{O}(1) +\right.}\boxed{\mathcal{O}(1)A + \mathcal{O}(1)\alpha_* r A'\Big]\,x} + \cancel{\mathcal{O}(1)x^2} - \boxed{\mathcal{O}(1)x^3} = 0$$



An example

$$A(r)=\begin{cases} \text{constant} & (r<r_S=0.01r_V) \\ (r_V/r)^3 & (r>r_S) \end{cases}$$



F=2, G=1, ξ_1=1.5, ξ_2=1, η=0.5, μ=1, ν=0.5, α_1=1.2, α_2=1, α_*=-0.25

Impact of $\nabla^2 \partial_t \epsilon$: linear growth

- Cosmological Poisson equation

$$\alpha = \left(1 + \mathcal{O}(1)\, \xi_t^2 \frac{\rho}{M_{\rm Pl}\Lambda^3}\right)^{1/2} a$$

$$\frac{\nabla^2 \Phi}{a^2 M_{\rm Pl}} = 4\pi G_{\rm eff}\, \rho \delta - 2\left(\frac{\dot{\alpha}}{\alpha} - H\right)\dot{\delta}$$

  - Time-dependent effective gravitational coupling
  - Additional friction from scalar field

- Evolution of density contrast

$$\ddot{\delta} + 2\frac{\dot{\alpha}}{\alpha}\dot{\delta} = 4\pi G_{\rm eff}\, \rho\, \delta$$

+ Energy-momentum conservation eq.

  - Beyond-term modifies the expansion rate felt by the density perturbations as well as $G_{\rm eff}$

Summary

- Effective Lagrangian on small scales is studied in the context of scalar-tensor theories beyond Horndeski

- New terms generates 3rd spatial derivatives in field eqs

  - ✓ Vainshtein screening mechanism operates outside the source and inside Vainshtein radius
  - ✓ $\Phi \neq \Psi$ *inside the source even in* $r<r_V$

- Linear growth of density perturbations is modified

  - ✓ G_{eff} and additional friction

Future

- Observational consequence of breaking of Vainshtein screening
  - Motion of a galaxy inside a cluster?
  - Modified structure of stars?

Thank you!

$$\widetilde{M}_{\rm Pl}^2\mathcal{F} \;\to\; \widetilde{M}_{\rm Pl}^2\mathcal{F}, \tag{A13}$$

$$\widetilde{M}_{\rm Pl}^2\mathcal{G} \;\to\; \widetilde{M}_{\rm Pl}^2\mathcal{G}, \tag{A14}$$

$$\begin{aligned}\widetilde{M}_{\rm Pl}\eta \;\to\;& \widetilde{M}_{\rm Pl}\eta + 4\left(10H^2X + 5\dot{H}X + 6H\dot{X}\right)F_4 + 4X\left(5H^2X + 2\dot{H}X + 9H\dot{X}\right)F_{4X} + 8HX^2\dot{X}F_{4XX}\\ &+12HX\dot{\phi}F_{4\phi} + 8HX^2\dot{\phi}F_{4\phi X} - 8HX\left(5H^2\dot{\phi} + 6\dot{H}\dot{\phi} + 10H\ddot{\phi}\right)F_5 - 8HX^2\left(2H^2\dot{\phi} + 2\dot{H}\dot{\phi} + 11H\ddot{\phi}\right)F_{5X}\\ &-16H^2X^3\ddot{\phi}F_{5XX} - 32H^2X^2F_{5\phi} - 16H^2X^3F_{5\phi X},\end{aligned} \tag{A15}$$

$$\widetilde{M}_{\rm Pl}\xi_1 \;\to\; \widetilde{M}_{\rm Pl}\xi_1 - 2HX\dot{\phi}\left(5F_4 + 2XF_{4X}\right) + 4H^2X^2\left(7F_5 + 2XF_{5X}\right), \tag{A16}$$

$$\widetilde{M}_{\rm Pl}\xi_2 \;\to\; \widetilde{M}_{\rm Pl}\xi_2, \tag{A17}$$

$$\frac{\widetilde{M}_{\rm Pl}\alpha_1}{\Lambda^3} \;\to\; \frac{\widetilde{M}_{\rm Pl}\alpha_1}{\Lambda^3} + X\left(5F_4 + 2XF_{4X}\right) - 2HX\dot{\phi}\left(7F_5 + 2XF_{5X}\right), \tag{A18}$$

$$\frac{\widetilde{M}_{\rm Pl}\alpha_2}{\Lambda^3} \;\to\; \frac{\widetilde{M}_{\rm Pl}\alpha_2}{\Lambda^3} + XF_4 - 2HX\dot{\phi}F_5, \tag{A19}$$

$$\frac{\widetilde{M}_{\rm Pl}\beta}{\Lambda^6} \;\to\; \frac{\widetilde{M}_{\rm Pl}\beta}{\Lambda^3} + \frac{2X}{3}\left(7F_5 + 2XF_{5X}\right), \tag{A20}$$

$$\begin{aligned}\frac{\mu}{\Lambda^3} \;\to\;& \frac{\mu}{\Lambda^3} - 2\left(\ddot{\phi} + 5H\dot{\phi}\right)F_4 - 5X\left(\ddot{\phi} + H\dot{\phi}\right)F_{4X} - 2X^2\ddot{\phi}F_{4XX} + XF_{4\phi} - 2X^2F_{4\phi X}\\ &+2\left(5H^2X + 2H\dot{X} + 14\dot{H}X\right)F_5 + 2\left(2H^2X + 2\dot{H}X + 11H\dot{X}\right)XF_{5X}\\ &+4HX^2\dot{X}F_{5XX} + 8HX\dot{\phi}F_{5\phi} + 4HX^2F_{5\phi X},\end{aligned} \tag{A21}$$

$$\frac{\nu}{\Lambda^3} \;\to\; \frac{\nu}{\Lambda^3} + 2F_4 + XF_{4X} - \frac{20}{3}\ddot{\phi}F_5 - \frac{22}{3}X\ddot{\phi}F_{5X} - \frac{4}{3}X^2\ddot{\phi}F_{5XX} - \frac{8}{3}XF_{5\phi} - \frac{4}{3}X^2F_{5\phi X}, \tag{A22}$$

$$\frac{\widetilde{M}_{\rm Pl}^2\gamma}{\Lambda^3} \;\to\; \frac{\widetilde{M}_{\rm Pl}^2\gamma}{\Lambda^3}. \tag{A23}$$