Recent developments in open Inflation

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Inflation with quantum tunneling
Part 1

INTRODUCTION
Inflation

- Almost scale invariant spectrum
- Highly Gaussian fluctuations
- Spatially quite flat

Excellent agreement with the prediction of the simplest inflationary model!

Today’s topic

Slow-roll inflation

Reheating

Field
It is quite possible that our part of the Universe appeared as a result of quantum tunneling after trapped in one of many metastable vacua.
Whole universe

Pocket universe

Eternal inflating region

Pocket universe
String theorists consider...

$O(10^{500})$ metastable dS states with different properties depending on vev of scalar fields, compactifications,...

The inflationary multiverse becomes divided into an exponentially large number of different exponentially large “pocket universes” with different laws of low-energy physics operating in each of them.
Is there any way to know what kind of neighborhood we live in?

Revisit “Open Inflation”!
Plan

1. Introduction (finish)

2. Open inflation

3. Recent developments
   3.1. Effect of rapid-rolling
   3.2. CMB asymmetry

4. Future prospects
Part 2

OPEN INFLATION
—BACKGROUND EVOLUTION—
Two possibilities

\[ |V_{\sigma\sigma}| \ll V_{\text{wall}} \rightarrow \text{Hawking-Moss tunneling} \]

\[ |V_{\sigma\sigma}| \gg V_{\text{wall}} \rightarrow \text{Coleman-de Luccia tunneling} \]
**Hawking-Moss (HM) tunneling**

- Tunneling to a **top of the barrier** due to the quantum fluctuations

  ✓ Too large density perturbations unless e-folds $>> 60$ [Linde(1995)]
  
  → If the final transition is HM, we can not see any deviation from the standard prediction of slow-roll inflation.

![Diagram showing tunneling to a top of the barrier due to quantum fluctuations](image)
Coleman-de Luccia (CDL) tunneling

- Tunneling mediated by an $O(4)$-symmetric solution of Euclidean Einstein-scalar equations.

- Reflecting the symmetry of a tunneling process, the region inside the bubble becomes an open FLRW universe!
O(4) symmetric CDL bounce solution

\[ ds^2 = dt^2_E + \cos^2 t_E (dr^2_E + \sin^2 r_E d\Omega^2) \]

Note that an O(4)-sym solution is expected to be most likely to occur (at least in the case of vacuum decay without gravity).

[Coleman+de Luccia (1980)]
Background evolution in CDL

\[ ds^2 = dt_E^2 + a^2(t_E) \left( dr_E^2 + \sin^2 r_E d\Omega^2 \right) \]

✓ Basic equations

\[
\begin{align*}
H^2 - \frac{1}{a_E^2} &= \frac{1}{3} \left( \frac{1}{2} \dot{\sigma}^2 - V(\sigma) \right) \\
\dot{H} + \frac{1}{a_E^2} &= -\frac{1}{2} \dot{\sigma}^2, \\
\ddot{\sigma} + 3H \dot{\sigma} - V' = 0.
\end{align*}
\]

✓ Boundary condition

\[
\begin{align*}
\dot{\sigma}(t_i) &= 0, \quad \dot{a}(t_i) = 1, \\
\dot{\sigma}(t_e) &= 0, \quad \dot{a}(t_e) = -1.
\end{align*}
\]
Background evolution in CDL

\[ \sigma(t_E) \]

\[ a(t_E) \]

\[ t_{E_i} \sim -\pi/2 \]

\[ t_{E} \sim \pi/2 \]

\[ -V(\sigma) \]

Bounce!
Geometry after CDL are obtained by analytic continuation of an CDL solution from Euclidean to Lorentzian:

\[-T^2 + R^2 = \frac{1}{H^2}\]

Hyperbolic

\[T_E^2 + R^2 = \frac{1}{H^2}\]

Spherical
\[
X^2 + Y^2 + Z^2 + U^2 + T_E^2 = H^{-2}
\]

\[
T_E = \cos[t_E] \cos[r_E]
\]
\[
U = \sin[t_E]
\]
\[
Z = \cos[t_E] \sin[r_E] \cos[\theta]
\]
\[
X = \cos[t_E] \sin[r_E] \sin[\theta] \cos[\phi]
\]
\[
Y = \cos[t_E] \sin[r_E] \sin[\theta] \sin[\phi]
\]

\[
T = \sinh[t_R] \cosh[r_R]
\]
\[
U = \cosh[t_R]
\]
\[
Z = \sinh[t_R] \sinh[r_R] \cos[\theta]
\]
\[
X = \sinh[t_R] \sinh[r_R] \sin[\theta] \cos[\phi]
\]
\[
Y = \sinh[t_R] \sinh[r_R] \sin[\theta] \sin[\phi]
\]

\[
T = \sinh[r_C] \cos[t_C]
\]
\[
U = \sin[t_C]
\]
\[
Z = \cosh[r_C] \cos[t_C] \cos[\theta]
\]
\[
X = \cosh[r_C] \cos[t_C] \sin[\theta] \cos[\phi]
\]
\[
Y = \cosh[r_C] \cos[t_C] \sin[\theta] \sin[\phi]
\]

Geometry after CDL

are obtained by analytic continuation of an CDL solution from Euclidean to Lorentzian:
Geometry after CDL

False vacuum

True vacuum

Open-slice of de Sitter [ O(3,1) sym ]

\[ ds^2 = -dt_R^2 + \sinh^2 t_R \left( dr_R^2 + \sinh^2 r_R d\Omega^2 \right) \]

Lorentzian

Euclidean

CDL solution [ O(4) sym ]

\[ ds^2 = dt_E^2 + \cos^2 t_E \left( dr_E^2 + \sin^2 r_E d\Omega^2 \right) \]
Geometry after CDL

**de Sitter Penrose-like diagram**

- **False vacuum**
- **True vacuum**
- **Bubble wall**
- **Bubble nucleation**

**Open-slice of de Sitter [ O(3,1) sym ]**

\[ ds^2 = -dt_R^2 + \sinh^2 t_R \left( dr_R^2 + \sinh^2 r_R \, d\Omega^2 \right) \]
Open Inflation

Trapped in one of metastable vacua → CDL tunneling

Reflecting the symmetry of a tunneling process, the region inside the bubble becomes open universe!

slow-roll inflation

reheating
Ex. 1: Simplest polynomial potential

\[ V(\sigma) = \frac{m^2}{2}\sigma^2 - \frac{\delta}{3}\sigma^3 + \frac{\lambda}{4}\sigma^4 \]

- \(|V_{,\sigma\sigma}| < 4H^2\): HM tunneling \[\text{[Linde (1999)]}\]
- The condition for CDL and subsequent slow-roll inflation are not easily satisfied at the same time!
Ex. 2: Simple two-field model

- Naturally/easily realized in the landscape

\[
\begin{align*}
\sigma & : \text{heavy field} \rightarrow \text{false vacuum decay} \\
\phi & : \text{light field} \rightarrow \text{starts rolling after FV decay}
\end{align*}
\]

\[
V(\sigma, \phi) = V_{\text{tunnel}}(\sigma) + m^2\phi^2/2
\]

- Too large perturbations from supercurvature mode of $\phi$ unless e-folds $>> 60$ [Sasaki+Tanaka(1996)]
Ex. 3: Multi-field tunneling and inflation

\[ |V_{,\sigma\sigma}| >> 4H^2 \]

- Single-field tunneling
- Multi-field tunneling
- Mass potential
- Light field: \( \phi \)
- Heavy field: \( \sigma \)
- Simplest polynomial potential
- Fast-rolling down → slow-roll inflation
Ex. 3: Tunneling rate $\sim \exp(-B)$

Multi-field dynamics tends to increase the tunneling-rate (?).

<table>
<thead>
<tr>
<th>$m_\phi$ $[m_{pl}]$</th>
<th>$\Delta \sigma$ $[m_{pl}]$</th>
<th>$\Delta \phi$ $[m_{pl}]$</th>
<th>$B$</th>
<th>$B_0$</th>
<th>$B_{HM}$</th>
<th>$\Delta B = B - B_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-6}$</td>
<td>1.91</td>
<td>$2.20 \times 10^{-10}$</td>
<td>12109.11</td>
<td>12109.11</td>
<td>12679.69</td>
<td>$</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>1.91</td>
<td>$2.20 \times 10^{-6}$</td>
<td>12108.10</td>
<td>12108.10</td>
<td>12678.65</td>
<td>$</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>1.91</td>
<td>$2.19 \times 10^{-4}$</td>
<td>12008.71</td>
<td>12008.71</td>
<td>12576.67</td>
<td>$</td>
</tr>
<tr>
<td>$5 \times 10^{-3}$</td>
<td>1.90</td>
<td>$5.34 \times 10^{-3}$</td>
<td>9975.05</td>
<td>9975.07</td>
<td>10484.43</td>
<td>$-0.02$</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>1.87</td>
<td>$1.97 \times 10^{-2}$</td>
<td>6322.66</td>
<td>6322.85</td>
<td>6691.28</td>
<td>$-0.19$</td>
</tr>
<tr>
<td>$2 \times 10^{-2}$</td>
<td>1.73</td>
<td>$6.00 \times 10^{-2}$</td>
<td>2188.07</td>
<td>2189.41</td>
<td>2305.67</td>
<td>$-1.35$</td>
</tr>
<tr>
<td>$3 \times 10^{-2}$</td>
<td>1.38</td>
<td>$8.73 \times 10^{-2}$</td>
<td>849.20</td>
<td>852.07</td>
<td>868.55</td>
<td>$-2.87$</td>
</tr>
<tr>
<td>$4 \times 10^{-2}$</td>
<td>0.49</td>
<td>$4.58 \times 10^{-2}$</td>
<td>372.15</td>
<td>376.39</td>
<td>372.28</td>
<td>$-4.25$</td>
</tr>
</tbody>
</table>
Quantization

**Step 1**

We need to find the reduced action that contains only the physical degree of freedom.

Ex) Simple scalar field

\[
S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{1}{2} m^2(\eta) \sigma^2 \right)
\]
Quantization

Step 2

We need to find a complete set of functions which obey the field equation and which are regular on the lower hemisphere.

A set of all modes which can be Klein-Gordon normalized on a Cauchy surface

\[ (\sigma_N, \sigma_M)_{KG} = -i \int_{\Sigma} d\Sigma^\mu g^{\mu\nu} \left\{ \sigma_N \partial_\nu \bar{\sigma}_M - (\partial_\nu \sigma_N) \bar{\sigma}_M \right\} = \delta_{NM} \]
We promote the physical degree of freedom to operator and expand it by mode functions \( \{\sigma_N, \overline{\sigma}_N\} \) as

\[
\hat{\sigma} = \sum_N \left[ \hat{a}_N \sigma_N + \hat{a}^\dagger_N \overline{\sigma}_N \right]
\]

\[
[\hat{a}_N, \hat{a}^\dagger_M] = \delta_{NM}
\]

Mode function \( \sigma_N \) plays the role of positive frequency functions!
Difficulty in quantization

The surfaces which respect the maximal sym. ($t_R=\text{const.}$) are not the Cauchy surface of the whole spacetime.

We need to work in the center region ($r_C=\text{const.}$), where the background configuration is spatially inhomogenous.

[For massive U(1) field, DY+Fujita+Mukohyama, 1402.2784]
Difficulty in quantization

The surfaces which respect the maximal sym. \((t_R=\text{const.})\) are not the Cauchy surface of the whole spacetime.

We need to work in the center region \((r_C=\text{const.})\), where the background configuration is spatially inhomogenous.

\[
ds^2 = -dt_R^2 + \sinh^2 t_R \left( dr_R^2 + \sinh^2 r_R d\Omega^2 \right)
\]

\[
ds^2 = dt_C^2 + \cos^2 t_C \left( -dr_C^2 + \cosh^2 r_C d\Omega^2 \right)
\]
(Discrete) supercurvature mode

There could appear a set of modes which have finite KG norms on Cauchy surfaces, but which cannot be quantized on the open chart because of the divergent KG norms on the open chart.

\[(\sigma_\mathcal{N}, \sigma_\mathcal{M})_{\text{KG}} = -i \int_{-\infty}^{\infty} d\eta_\mathcal{C} \int d\Omega a^2(\eta_\mathcal{C}) \left\{ \sigma_\mathcal{N} \partial_{r_\mathcal{C}} \bar{\sigma}_\mathcal{M} - (\partial_{r_\mathcal{C}} \sigma_\mathcal{N}) \bar{\sigma}_\mathcal{M} \right\} = \delta_{\mathcal{N}\mathcal{M}} \]
(Discrete) supercurvature mode

There could appear a set of modes which have finite KG norms on Cauchy surfaces, but which **cannot be quantized on the open chart** because of the divergent KG norms on the open chart.

- Canonical scalar field
  [Sasaki+Tanaka+Yamamoto (1995)]
  \( \times \) (for \( m > 3H/2 \))

- Tensor perturbations
  [Tanaka+Sasaki (1997)]
  \( \times \) (for \( m \leq 3H/2 \))

- U(1) gauge field
  [DY+Fujita+Mukohyama, 1402.2784]
  \( \times \)

- Metric pert.+scalar field
  [Garriga+Montes+Sasaki+Tanaka(1999)]
  \( ? \) (model-depend)
Evolution inside bubble

Log $L$ vs Log $a$

Curvature-dominated era

Inflationary era

Supercurvature mode

Normal mode

Curvature scale

$H^{-1}$

After bubble nucleation
Power spectrum for scalar/tensor

\[ k^3 P(k) / 2\pi^2 \]

- **Scalar suppression**
  - [Linde+Sasaki+Tanaka(1999)]
  - [White+Zhang+Sasaki(2014)]

- **Tensor enhancement**
  - [DY+Linde+Naruko+Sasaki+Tanaka(2011)]

- **Supercurvature mode**

\[ k^3 P(k) / 2\pi^2 \]

- **Tensor with rapid-rolling**

- **Scalar**
  - \( \sim 2k_{\text{curv}} \)

- **Tensor**
  - \( \sim 2k_{\text{curv}} \)

(若干の適当さを含みます)
Part 3

RECENT DEVELOPMENTS 1
— SIGNALS FROM STRINGY INFLATION —
## Classification for open inflation

<table>
<thead>
<tr>
<th>Inflation driver (m&lt;H)</th>
<th>Tunneling driver (m&gt;H)</th>
<th>Curvature pert. ζ</th>
<th>Supercurvature mode?</th>
<th>note</th>
</tr>
</thead>
</table>
| σ                      | σ                      | σ                 | σ \rightarrow \bigcirc/\times | • Linde(1998)  
• Linde+Sasaki+Tanaka(1999)  
• Garriga+Montes+Sasaki+Tanaka(1999)  
• DY+(2011) |
| σ                      | ρ                      | σ                 | σ \rightarrow \bigcirc | • Linde+Mezhlumian(1995)  
• Sasaki+Tanaka(1996)  
[• tunneling sol: Sugimura+DY+Sasaki(2011) ] |
| ϕ                      | ρ                      | σ                 | σ \rightarrow \bigcirc | • Kanno+Sasaki+Tanaka(2013)  
• Brnes+Domenech+Sasaki+Takahashi(2016)  
[• Sugimura+DY+Sasaki(2012) ] |
Requirements for a single-field stringy model

The universe will most likely tunnel to a point where the energy scale is still very high and have a large hierarchy between energy scales.

\[ V_{\text{landscape}} = \mathcal{O}(M_{\text{Pl}}^4) \gg V_{\text{inf}} \]

Ex.) Stringy inflation \( V_{\text{inf}} = \mathcal{O}(10^{-30}M_{\text{Pl}}^4) \)

[Kallosh+Linde(2014)]
Requirements for a single-field stringy model

Slow-roll inflation does not start immediately after the CDL tunneling, and there must be some intermediate stage of rapid rolling down.

\[ |V_{,\sigma\sigma}| \gg V/M_{Pl}^2 \]

Tensor-type perturbations may not be suppressed at all!

\[ \langle |h|^2 \rangle \approx \frac{2}{M_{pl}^2} \left( \frac{H}{2\pi} \right)^2 \]

Memory of false vacuum may remain in the perturbations on the curvature scale!

✓ Note: Scalar perturbations may be suppressed by the velocity during RAPID ROLL phase...

\[ \langle |\mathcal{R}^2| \rangle \approx \left( \frac{H^2}{2\pi \dot{\phi}} \right)^2 \]
Primordial tensor spectrum

Keeps the memory of the high energy density in large angular scales, and can be strongly red-tilted.

A toy model:

$$V(\phi) \propto \phi^2 \exp\left[\left(\phi^2 - \phi_R^2\right) / \phi_R^2\right]$$

Normalized tensor spectrum

Wave number normalized by curvature scale

Scale-inv. spectrum
Primordial tensor spectrum

Keeps the memory of the high energy density in large angular scales, and can be strongly red-tilted.

A toy model:
\[ V(\phi) \propto \phi^2 \exp\left[\left(\phi^2 - \phi_R^2\right)/\phi_R^2\right] \]
Part 4

ASYMMETRY FROM OPEN INFLATION
Polarizations from open inflation

- Spatially openness: $\Omega_K = 10^{-2} - 10^{-4}$
- A complete set of mode functions
  - (Continuous) normal modes
  - (Discrete) supercurvature modes

Implement in CAMB code

CMB temperature + polarizations

Large enhancement due to rapid-rolling?
CMB anomalies

\[ \Delta T(n) = (1 + A \mathbf{p} \cdot n) \Delta T_{\text{iso}}(n) \]

\[ A = 0.07 \pm 0.02 \]
Modulation due to supercurvature

We will treat the supercurvature mode as a nonstochastic quantity and we can only observe one realization in our Hubble patch.

The modulation of the continuous spectrum

[see also Kanno+Sasaki+Tanaka(2013), Bernes+Domenech+Sasaki+Takahashi(2016)]

[DY+Zarei+Hirouzjahi+Ohta+Naruko+Yamaguchi, work in progress]
Modulation due to supercurvature

We will treat the supercurvature mode as a nonstochastic quantity and we can only observe one realization in our Hubble patch.

The modulation of the continuous spectrum

\[ \mathcal{R}_c(x) = \mathcal{N} \left| \sigma_{bg} - \mathcal{N} \left| \sigma_{bg} + \Delta \sigma(x) - \mathcal{N} \left| \sigma_{bg} + \Delta \sigma(x) \right| \delta \sigma(x) - \frac{1}{2} \mathcal{N} \sigma \delta \sigma^2(x) + \cdots \right. \]

Spatial modulation due to SC mode!

- Power asymmetry: \( A = \Delta P_R / 2P_R \)
- Quadrupole: \( \sigma_{20} \)

[see also Kanno+Sasaki+Tanaka(2013), Bernes+Domenech+Sasaki+Takahashi(2016)]

[DY+Zarei+Hirouzjahi+Ohta+Naruko+Yamaguchi, work in progress]
Modulation due to supercurvature

[see also Kanno+Sasaki+Tanaka(2013), Bernes+Domenech+Sasaki+Takahashi(2016)]

We will treat the supercurvature mode as a nonstochastic quantity and we can only observe one realization in our Hubble patch.

The modulation of the continuous spectrum

Hemispherical power asymmetry!
We will treat the supercurvature mode as a nonstochastic quantity and we can only observe one realization in our Hubble patch.

Question...

✓ Does the supercurvature mode with the large amplitude really exist in the realistic open inflationary scenario?
Future prospects

- Multifield tunneling and supercurvature mode

✓ Background solution [Sugimura+DY+Sasaki (2011)]

✓ Quantization including the effect of the metric perturbations should be taken into account.

✓ would rescue the simplest two field model [Tanaka+Sasaki(1996)]

\[ L = \left( \frac{M_{pl}^2}{2} \right) R - G_{ab}(\varphi) d\varphi^a \cdot d\varphi^b / 2 - V(\varphi) \]

supercurvature modes?
Future prospects

Higher-order correlations

- Bispectrum from scalar normal modes in exact dS
  [Sugimura+Komatsu (2013)]

\[
B_{\zeta}^{\text{subcurv}} \approx B_{\zeta}^{\text{usual}} + B_{\zeta}^{\text{NBD}} \quad \text{(squeezed limit, subcurvature limit)}
\]

\[
\begin{align*}
B_{\zeta}^{\text{usual}} & \rightarrow (1-n_s)P_{\zeta}(k_{\text{long}})P_{\zeta}(k_{\text{short}}) \\
B_{\zeta}^{\text{NBD}} & \rightarrow \left( k_{\text{short}}/k_{\text{long}} \right) \exp(-\pi k_{\text{short}}) P_{\zeta}(k_{\text{long}})P_{\zeta}(k_{\text{short}})
\end{align*}
\]

- Extension to the dynamical background is needed.

- Contributions from supercurvature modes@\(k \sim k_{\text{superlong}}\)?
Summary

We are already testing the model of inflation in the context of cosmic/string landscape!